SPIRAL
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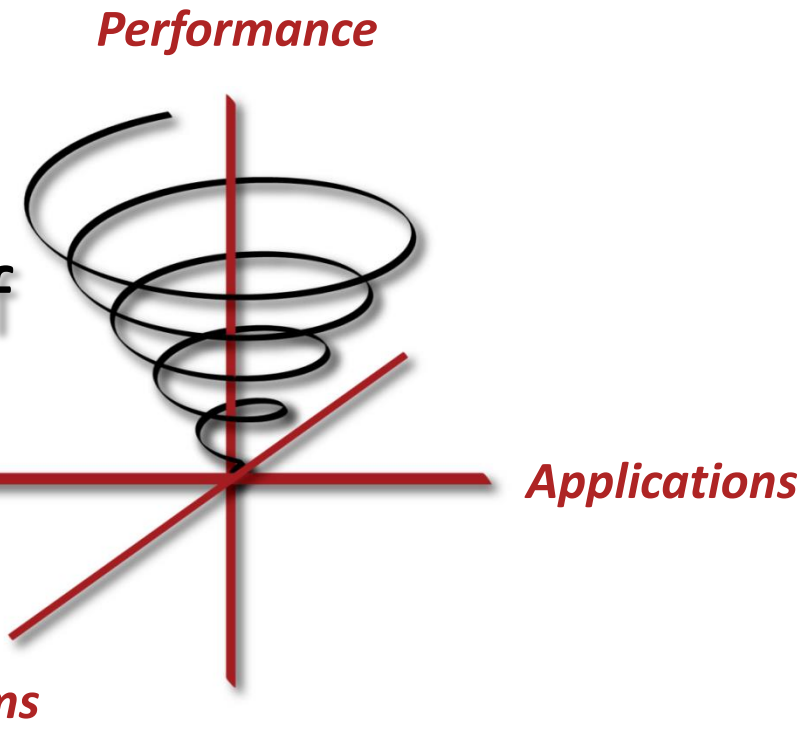
Spiral

Computer Generation of Performance Libraries



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Carnegie Mellon

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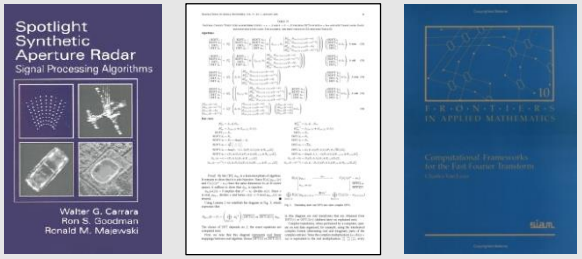
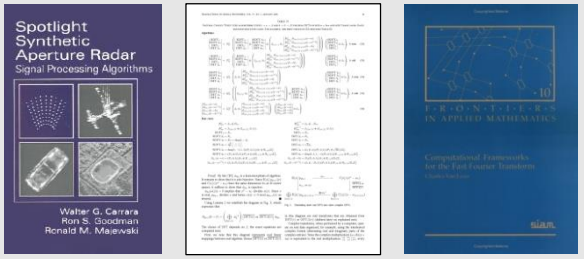
What is Spiral?

Traditionally



High performance library optimized for given platform

Spiral Approach



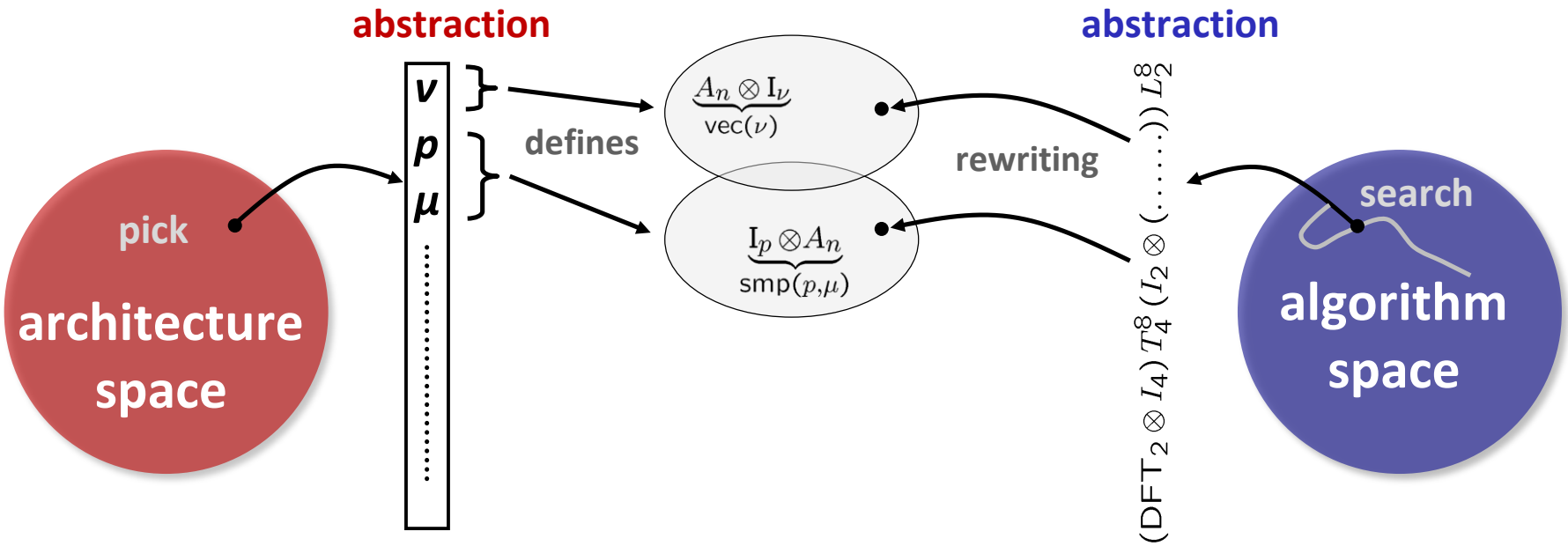
Spiral

High performance library optimized for given platform

Comparable performance

Main Idea: Program Generation

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

optimization

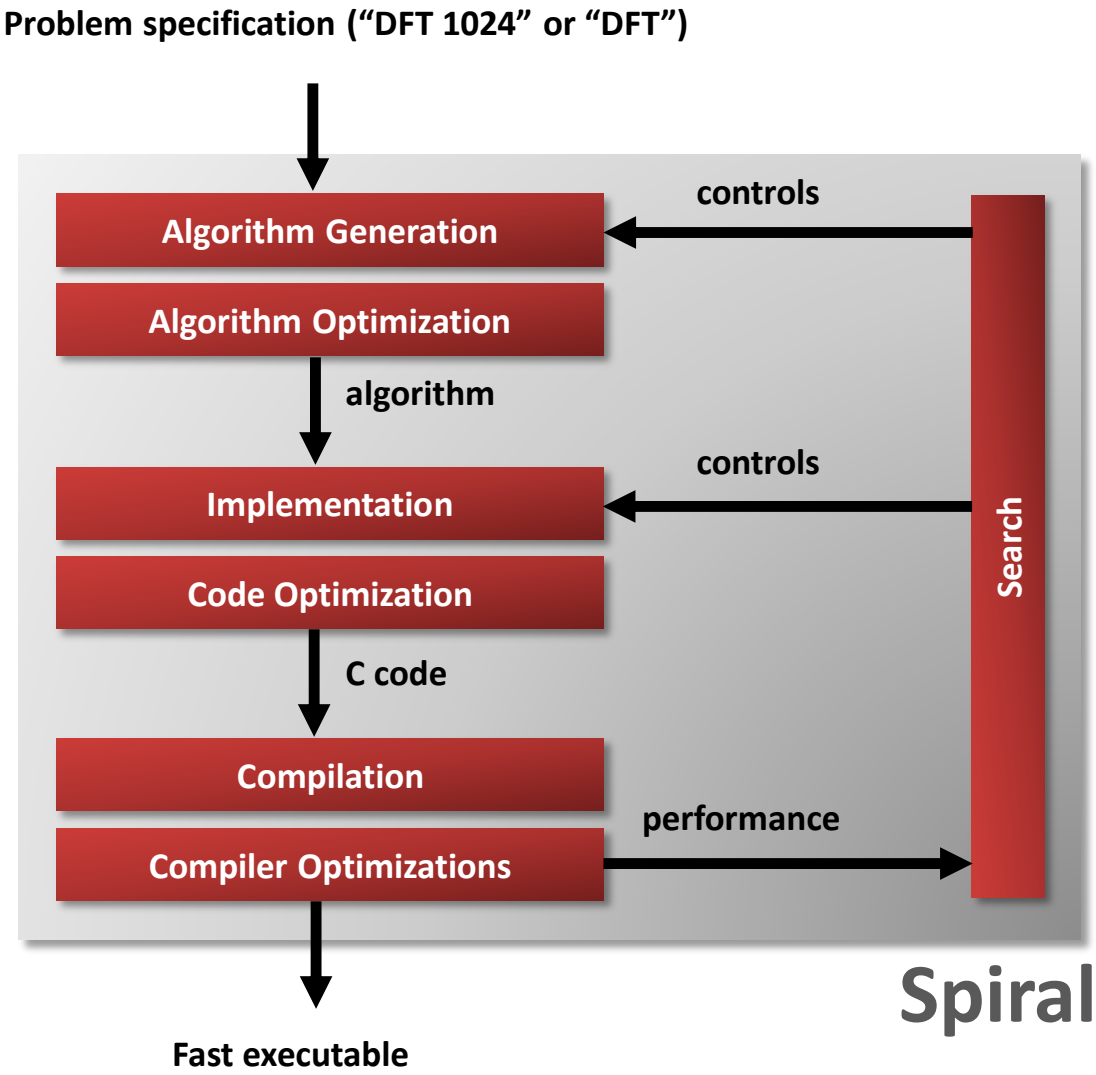
Kernel:
problem size,
algorithm choice

How Spiral Works

Complete automation of the implementation and optimization task

Basic ideas:

- ***Declarative representation*** of algorithms
- ***Rewriting systems*** to generate and optimize algorithms at a high level of abstraction



Algorithms: Rules in Domain Specific Language

Linear Transforms

$$\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m)Q_n, \quad n = km,\ \gcd(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})D_p(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\, \mathrm{L}^n_m(\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\qquad \cdot(\mathrm{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m)\left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right)\right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathrm{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathrm{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathrm{J}_2\, \mathrm{R}_{13\pi/8}
\end{aligned}$$

Viterbi Decoding

$$\begin{aligned}
\underbrace{\mathbf{Vit}}_{\mathrm{vec}(v)} &\rightarrow \underbrace{\left(\prod (L \times I) \circ (I \otimes C)\right) \circ Id}_{\mathrm{vec}(v)} \\
&\rightarrow \left(\prod \underbrace{(L \times I) \circ (I \otimes C)}_{\mathrm{vec}(v)}\right) \circ Id \\
\pounds &\rightarrow \left(\prod (L \otimes I_v \times I) \circ (I \otimes C \otimes I_v) \circ (\vec{L} \times I)\right) \circ Id \\
&\rightarrow \prod (L \otimes I_v \times I) \circ (I \otimes (B \otimes I_v)) \circ (\vec{L} \times I)
\end{aligned}$$

Matrix-Matrix Multiplication

$$\begin{aligned}
&\mathrm{MMM}_{1,1,1} \rightarrow (\cdot)_1 \\
&\mathrm{MMM}_{m,n,k} \rightarrow (\otimes)_{m/m_b \times 1} \otimes \mathrm{MMM}_{m_b,n,k} \\
&\mathrm{MMM}_{m,n,k} \rightarrow \mathrm{MMM}_{m,nb,k} \otimes (\otimes)_{1 \times n/nb} \\
&\mathrm{MMM}_{m,n,k} \rightarrow ((\Sigma_{k/k_b} \circ (\cdot)_{k/k_b}) \otimes \mathrm{MMM}_{m,n,k_b}) \circ \\
&\qquad\qquad ((L^{mk/k_b}_{k/k_b} \otimes I_{k_b}) \times I_{kn}) \\
&\mathrm{MMM}_{m,n,k} \rightarrow (L^{mn/n_b}_m \otimes I_{n_b}) \circ \\
&\qquad ((\otimes)_{1 \times n/n_b} \otimes \mathrm{MMM}_{m,n_b,k}) \circ \\
&\qquad\qquad (I_{km} \times (L^{kn/n_b}_{n/n_b} \otimes I_{n_b}))
\end{aligned}$$

Synthetic Aperture Radar (SAR)

$$\begin{aligned}
\mathrm{SAR}_{k \times m \rightarrow n \times n} &\rightarrow \mathrm{DFT}_{n \times n} \circ \mathrm{Interp}_{k \times m \rightarrow n \times n} \\
\mathrm{DFT}_{n \times n} &\rightarrow (\mathrm{DFT}_n \otimes \mathrm{I}_n) \circ (\mathrm{I}_n \otimes \mathrm{DFT}_n) \\
\mathrm{Interp}_{k \times m \rightarrow n \times n} &\rightarrow (\mathrm{Interp}_{k \rightarrow n} \otimes_i \mathrm{I}_n) \circ (\mathrm{I}_k \otimes_i \mathrm{Interp}_{m \rightarrow n}) \\
\mathrm{Interp}_{r \rightarrow s} &\rightarrow \left(\bigoplus_{i=0}^{n-2} \mathrm{InterpSeg}_k\right) \oplus \mathrm{InterpSegPruned}_{k,\ell} \\
\mathrm{InterpSeg}_k &\rightarrow \mathrm{G}^{u \cdot n \rightarrow k}_f \circ \mathrm{iPrunedDFT}_{n \rightarrow u \cdot n} \circ \left(\frac{1}{n}\right) \circ \mathrm{DFT}_n
\end{aligned}$$

One Approach for all Types of Parallelism

- **Multithreading** (Multicore)
- **Vector SIMD** (SSE, VMX/Altivec,…)
- **Message Passing** (Clusters, MPP)
- **Streaming/multibuffering** (Cell)
- **Graphics Processors** (GPUs)
- **Gate-level parallelism** (FPGA)
- **HW/SW partitioning** (CPU + FPGA)

$$\mathrm{I}_p \otimes_{\|} A_{\mu n}, \quad \mathbf{L}^{mn}_{m} \bar{\otimes} \mathrm{I}_\mu$$

$$A \hat{\otimes} \mathrm{I}_\nu \quad \underbrace{\mathbf{L}^{2\nu}_{2}}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{2\nu}_{\nu}}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{\nu^2}_{\nu}}_{\text{isa}}$$

$$\mathrm{I}_p \otimes_{\|} A_n, \quad \underbrace{\mathbf{L}^{p^2}_{p} \bar{\otimes} \mathrm{I}_{n/p^2}}_{\text{all-to-all}}$$

$$\mathrm{I}_n \otimes_2 A_{\mu n}, \quad \mathbf{L}^{mn}_{m} \bar{\otimes} \mathrm{I}_\mu$$

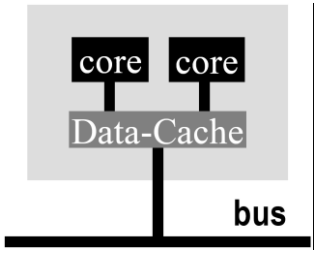
$$\prod_{i=0}^{n-1} A_i, \quad A_n \hat{\otimes} \mathrm{I}_w, \quad P_n \otimes Q_w$$

$$\prod_{i=0}^{n-1^{\mathsf{ir}}} A, \quad \mathrm{I}_s \tilde{\otimes} A, \quad \underbrace{\mathbf{L}^{m}_{n}}_{\text{bram}}$$

$$\underbrace{A_1}_{\text{fpga}}, \quad \underbrace{A_2}_{\text{fpga}}, \quad \underbrace{A_3}_{\text{fpga}}, \quad \underbrace{A_4}_{\text{fpga}}$$

Example: Code Generation for Multicore CPUs

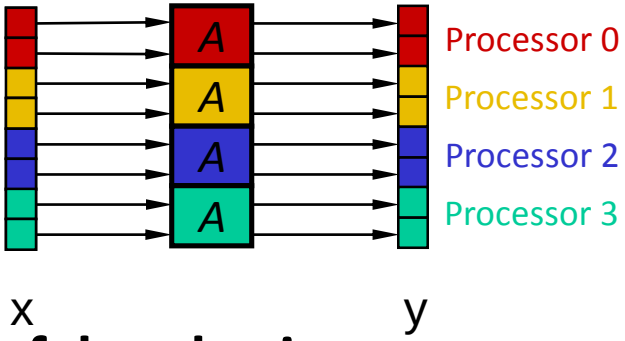
- **Hardware abstraction: shared cache with cache lines**



$$\underbrace{A_{k\times m\to n}}_{\mathrm{smp}(p,\mu)}$$

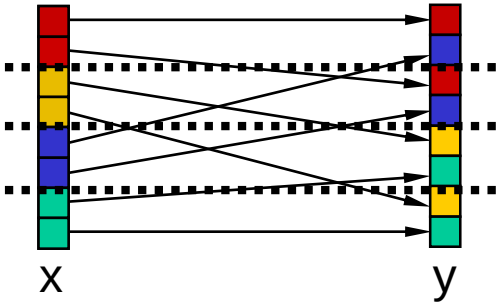
- **Tensor product: embarrassingly parallel operator**

$$y = \left(\mathrm{I}_p \otimes A\right)(x)$$



- **Permutation: problematic; may produce false sharing**

$$y = \mathrm{L}^{8}_{4}(x)$$



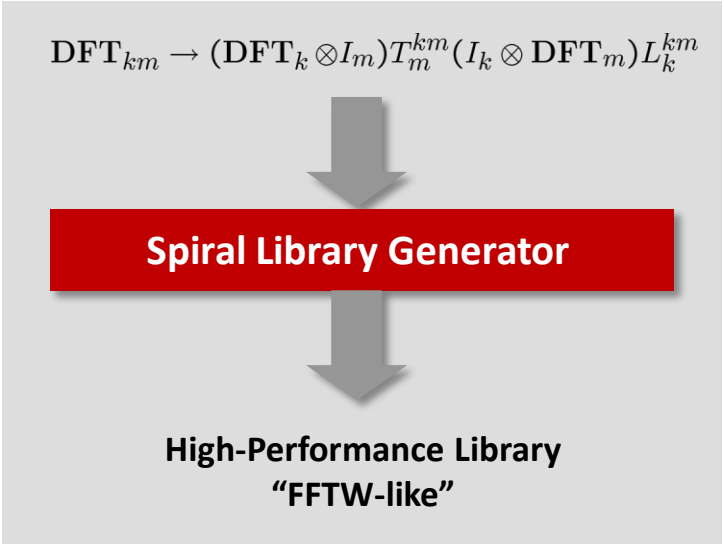
Spiral: Meta-Tool to Autotuning Libraries

Input:

- **Transform**: DFT_n
- **Algorithms**: $\mathrm{DFT}_{km} \rightarrow (\mathrm{DFT}_k \otimes I_m) T^{km}_m (I_k \otimes \mathrm{DFT}_m) L^{km}_k$
  $\mathrm{DFT}_2 \rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
- **Vectorization**: 2-way SSE
- **Threading**: Yes

Output:

- Optimized library (10,000 lines of C++)
- For general input size
  (**not** collection of fixed sizes)
- Vectorized
- Multithreaded
- With runtime adaptation mechanism
- Performance competitive with hand-written code



Verification and Testing

- **Verify algorithms symbolically**

$$\mathrm{DFT}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \stackrel{?}{=} (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\, \mathrm{T}^4_2 (\mathrm{I}_2 \otimes \mathrm{DFT}_2)\, \mathrm{L}^4_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & j \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Check rules through verification of instances**

$$\mathrm{I}_m \otimes A_n \rightarrow \mathrm{L}^{mn}_m (A_n \otimes \mathrm{I}_m)\, \mathrm{L}^{mn}_n$$

$$\mathrm{I}_2 \otimes \mathrm{DFT}_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \stackrel{?}{=} \mathrm{L}^4_2 (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\, \mathrm{L}^4_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Check code empirically**

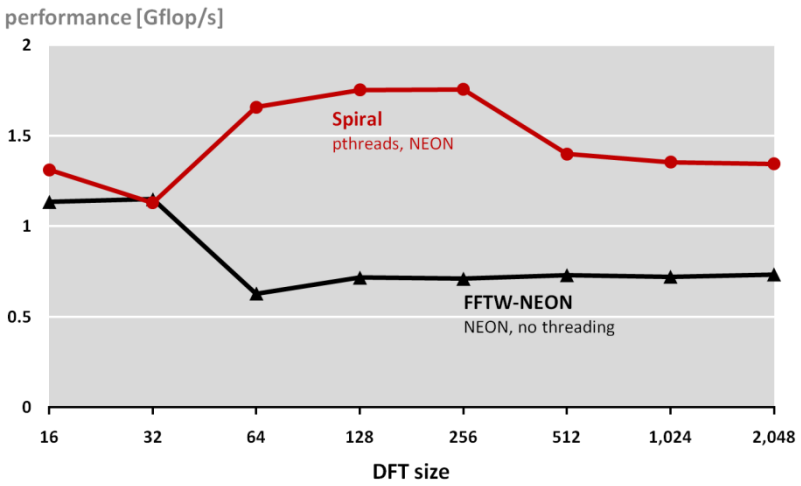
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \stackrel{?}{=} \texttt{DFT4([0,1,0,0])}$$

```
DFT4([0.1,1.77,2.28,-55.3]) = ? DFT4_rnd([0.1,1.77,2.28,-55.3]))
```

Range: Cell Phone To Supercomputer

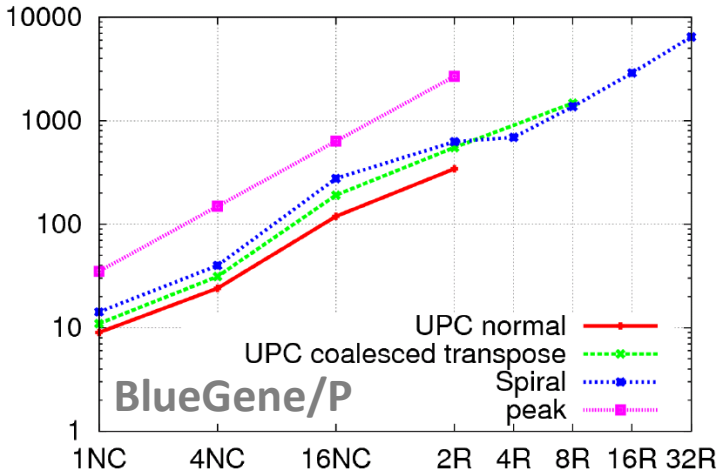
DFT on Samsung Galaxy S II

Dual-core 1.2 GHz Cortex-A9 with NEON ISA



Global FFT (1D FFT, HPC Challenge)

performance [Gflop/s]



6.4 Tflop/s

Samsung i9100 Galaxy S II

Dual-core ARM at 1.2GHz with NEON ISA

SIMD vectorization + multi-threading

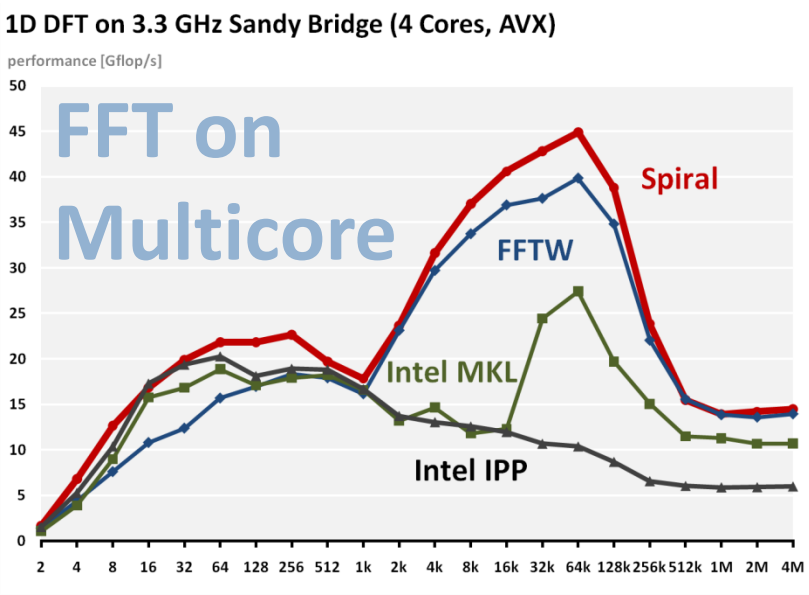
BlueGene/P at Argonne National Laboratory

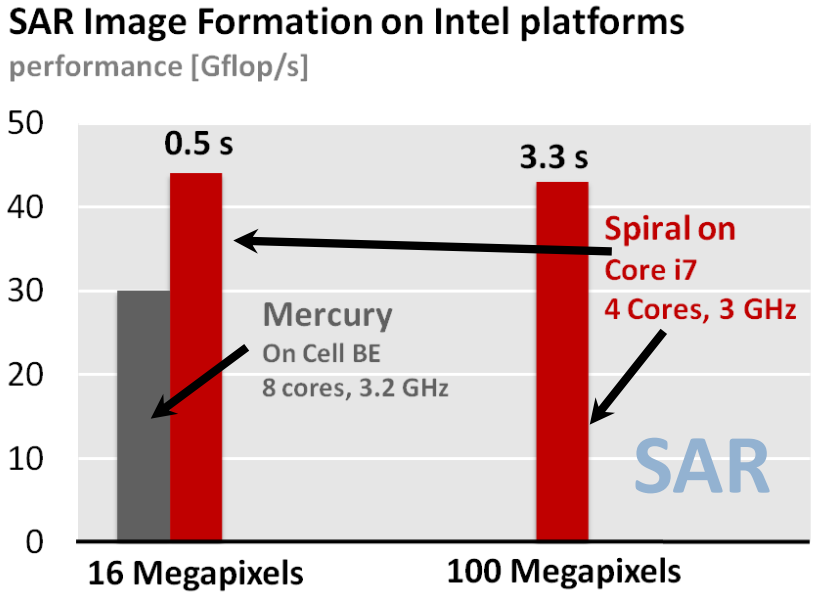
128k cores (quad-core CPUs) at 850 MHz

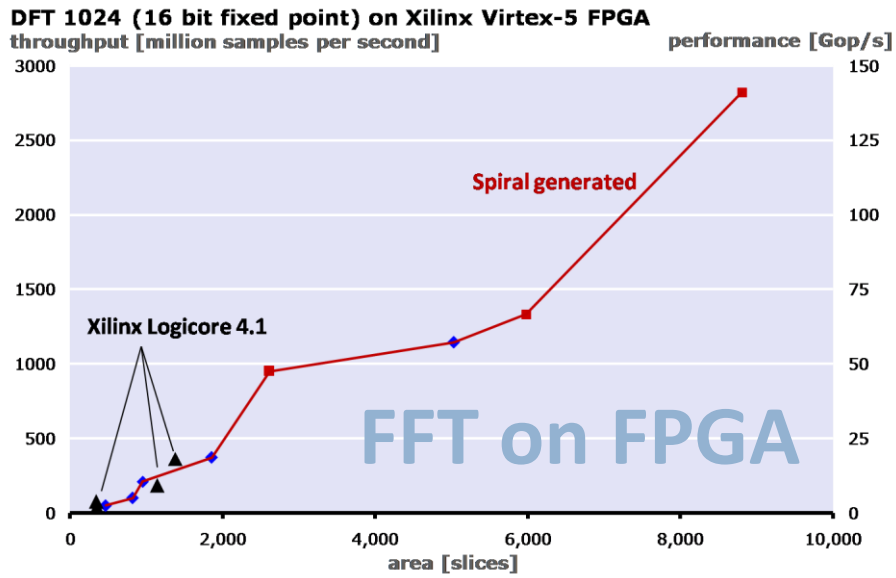
SIMD vectorization + multi-threading + MPI

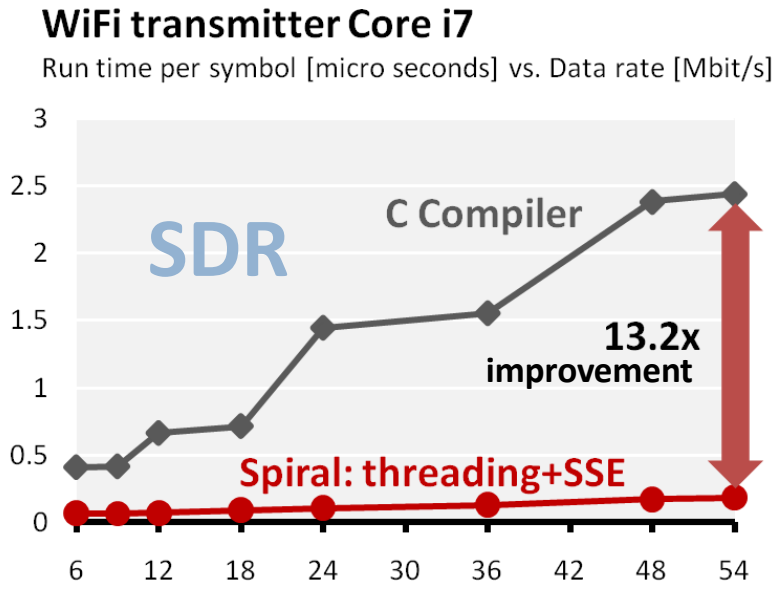
G. Almási, B. Dalton, L. L. Hu, F. Franchetti, Y. Liu, A. Sidelnik, T. Spelce, I. G. Tānase, E. Tiotto, Y. Voronenko, X. Xue:
2010 IBM HPC Challenge Class II Submission. **Winner of the 2010 HPC Challenge Class II Award (Most Productive System).**

More Results: Spiral Outperforms Humans









More Information:

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