FFTX and SpectralPACK:
A First Look

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EXASCALE COMPUTING PROJECT

Have You Ever Wondered About This?

Numerical Linear Algebra

LAPACK
ScaLAPACK
LU factorization
Eigensolves
SVD

BLAS, BLACS
BLAS-1
BLAS-2
BLAS-3

Spectral Algorithms

Convolution
Correlation
Upsampling
Poisson solver
…

?

FFTW
DFT, RDFT
1D, 2D, 3D,…
batch

No LAPACK equivalent for spectral methods

- **Medium size 1D FFT (1k—10k data points) is most common library call**
  applications break down 3D problems themselves and then call the 1D FFT library
- **Higher level FFT calls rarely used**
  FFTW *guru* interface is powerful but hard to used, leading to performance loss
- **Low arithmetic intensity and variation of FFT use make library approach hard**
  Algorithm specific decompositions and FFT calls intertwined with non-FFT code

It Is Worse Than It Seems

Issue 1: 1D FFTW call is standard kernel for many applications

- **Parallel libraries and applications reduce to 1D FFTW call**
  P3DFFT, QBox, PS/DNS, CPMD, HACC,…
- **But: Reduction to 1D FFT leaves performance on the table**
  1D FFT is too low level of abstraction for modern HPC machines

Issue 2: FFTW is dominant but slowly becoming obsolete

FFTW

- **FFTW is supported by modern languages and environments**
  Python, Matlab,…
- **Vendor libraries support (parts of) the FFTW 3.X interface**
  Intel MKL, IBM ESSL, AMD ACML (end-of-life), Nvidia cuFFT, Cray LibSci/CRAFFT
- **But: FFTW 2.X/3.X reference implementation is dormant**
  Only minor updates/bug fixes since 2004, no native support for GPUs, well-known issues with MPI version

Risk: loss of high performance FFT standard library

FFTX: The FFTW Revamp for ExaScale

EXASCALE COMPUTING PROJECT

Modernized FFTW-style interface

- **Backwards compatible to FFTW 2.X and 3.X**
  old code runs unmodified and gains substantially but not fully
- **Small number of new features**
  futures/delayed execution, offloading, data placement, callback kernels
- **Reference library implementation and bindings to vendor libraries**
  library-only reference implementation for ease of development

Code generation backend using SPIRAL

- **Library/application kernels are interpreted as specifications in DSL**
  extract semantics from source code and known library semantics
- **Compilation and advanced performance optimization**
  cross-call and cross library optimization, accelerator off-loading,…
- **Fine control over resource expenditure of optimization**
  compile-time, initialization-time, invocation time, optimization resources

FFTX and SpectralPACK

Numerical Linear Algebra		Spectral Algorithms
LAPACK LU factorization Eigensolves SVD …	≈	**SpectralPACK** Convolution Correlation Upsampling Poisson solver …
BLAS BLAS-1 BLAS-2 BLAS-3		**FFTX** DFT, RDFT 1D, 2D, 3D,… batch

Define the LAPACK equivalent for spectral algorithms

- **Define FFTX as the BLAS equivalent**
  provide user FFT functionality as well as algorithm building blocks
- **Define class of numerical algorithms to be supported by SpectralPACK**
  PDE solver classes (Green's function, sparse in normal/k space,…), signal processing,…
- **Library front-end, code generation and vendor library back-end**
  mirror concepts from FFTX layer

FFTX and SpectralPACK solve the "spectral motif" long term

Example: Poisson's Equation in Free Space

Partial differential equation (PDE)

$$\Delta(\Phi) = \rho$$

$$\rho : \mathbb{R}^3 \to \mathbb{R}$$

$$D = \mathrm{supp}(\rho) \subset \mathbb{R}^3$$

Poisson's equation. Δ is the Laplace operator

Solution characterization

$$\Phi : \mathbb{R}^3 \to \mathbb{R}$$

$$\Phi(\vec{x}) = \frac{Q}{4\pi||\vec{x}||} + o\left(\frac{1}{||\vec{x}||}\right) \quad \text{as } ||\vec{x}|| \to \infty$$

$$Q = \int_D \rho d\vec{x}$$

Approach: Green's function

$$\Phi(\vec{x}) = \int_D G(\vec{x} - \vec{y})\rho(\vec{y})d\vec{y} \equiv (G * \rho)(\vec{x}), \quad G(\vec{x}) = \frac{1}{4\pi||\vec{x}||_2}$$

Solution: $\phi(.)$ = convolution of RHS $\rho(.)$ with Green's function $G(.)$. Efficient through FFTs (frequency domain)

Method of Local Corrections (MLC)

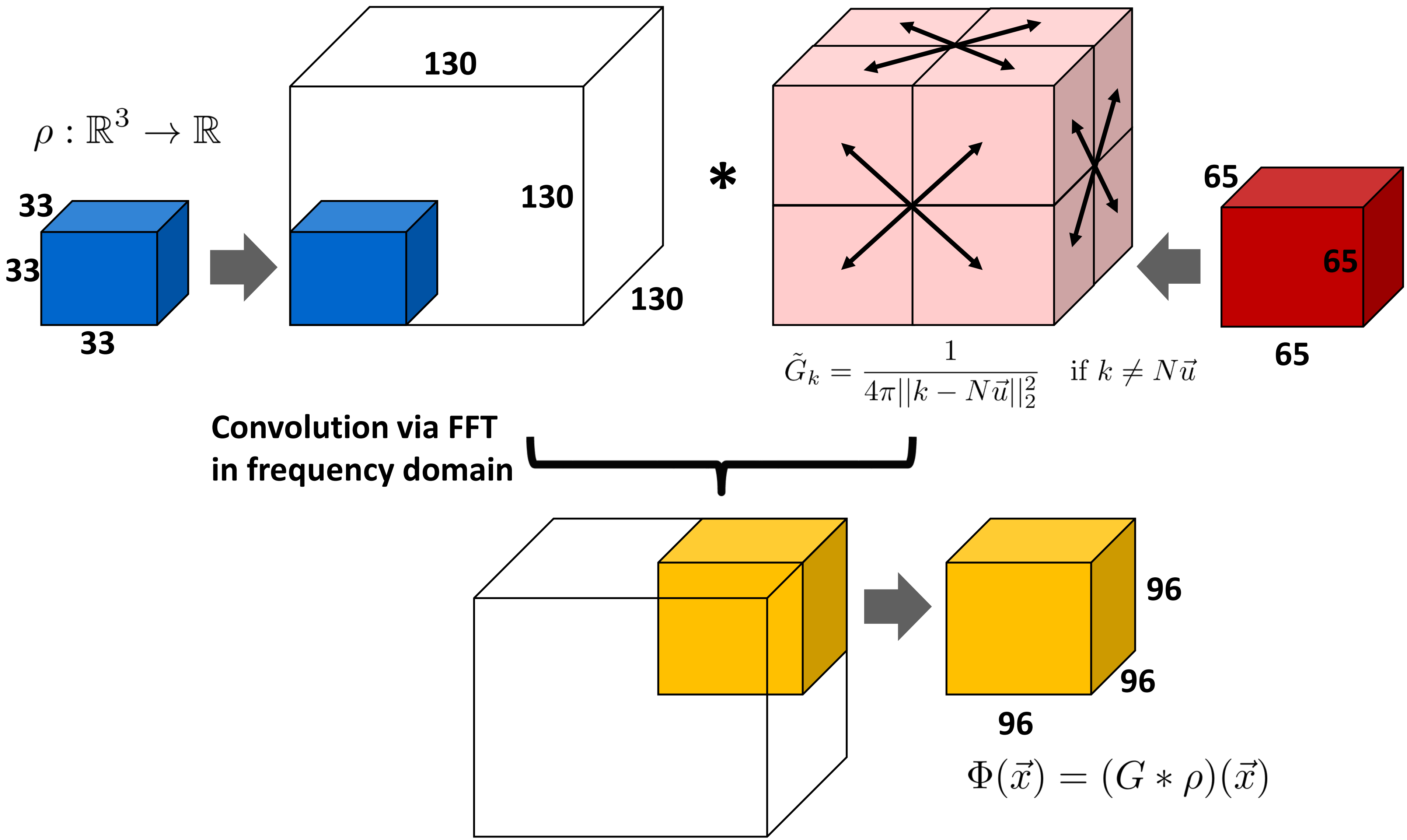
$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

Green's function kernel in frequency domain

P. McCorquodale, P. Colella, G. T. Balls, and S. B. Baden, "A local corrections algorithm for solving Poisson's equation in three dimensions," vol. 2, 10 2006.

C. R. Anderson, "A method of local corrections for computing the velocity field due to a distribution of vortex blobs," Journal of Computational Physics, vol. 62, no. 1, pp. 111–123, 1986.

Algorithm: Hockney Free Space Convolution



Hockney: Convolution + problem specific zero padding and output subset

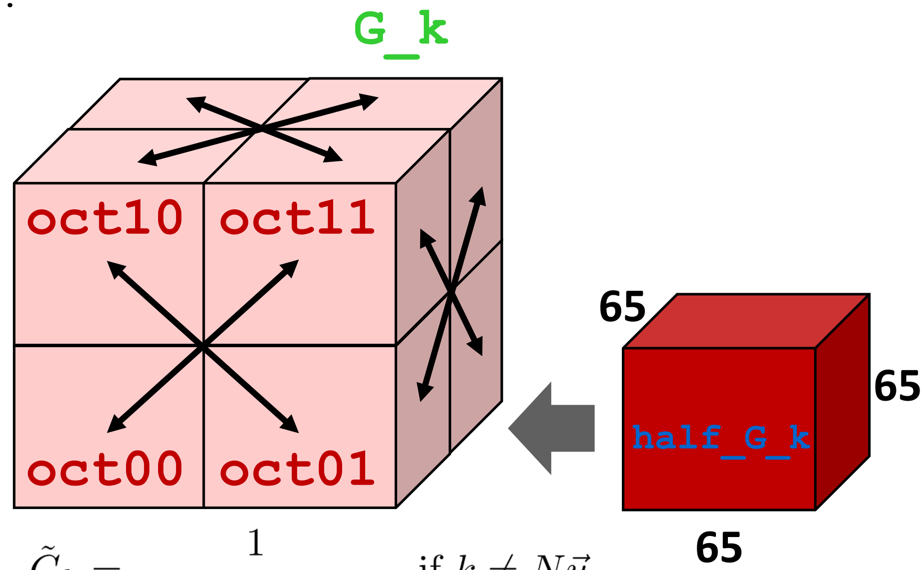
FFTX C Code: Hockney Free Space Convolution

```
fftx_plan pruned_real_convolution_plan(fftx_real *in, fftx_real *out, fftx_complex *symbol,
        int n, int n_in, int n_out, int n_freq) {
    int rank = 3,
    batch_rank = 0,
    ...
    fftx_plan plans[5];
    fftx_plan p;

    tmp1 = fftx_create_zero_temp_real(rank, &padded_dims);

    plans[0] = fftx_plan_guru_copy_real(rank, &in_dimx, in, tmp1, MY_FFTX_MODE_SUB);

    tmp2 = fftx_create_temp_complex(rank, &freq_dims);
    plans[1] = fftx_plan_guru_dft_r2c(rank, &padded_dims, batch_rank,
        &batch_dims, tmp1, tmp2, MY_FFTX_MODE_SUB);

    tmp3 = fftx_create_temp_complex(rank, &freq_dims);
    plans[2] = fftx_plan_guru_pointwise_c2c(rank, &freq_dimx, batch_rank, &batch_dimx,
        tmp2, tmp3, symbol, (fftx_callback)complex_scaling,
        MY_FFTX_MODE_SUB | FFTX_PW_POINTWISE);

    tmp4 = fftx_create_temp_real(rank, &padded_dims);
    plans[3] = fftx_plan_guru_dft_c2r(rank, &padded_dims, batch_rank,
        &batch_dims, tmp3, tmp4, MY_FFTX_MODE_SUB);

    plans[4] = fftx_plan_guru_copy_real(rank, &out_dimx, tmp4, out, MY_FFTX_MODE_SUB);

    p = fftx_plan_compose(numsubplans, plans, MY_FFTX_MODE_TOP);

    return p;
}
```

Looks like FFTW calls, but is a specification for SPIRAL

FFTX C Code: Describing The Hockney Symmetry

```
// FFTX data access descriptors.
// Access is to four octants of a symmetric cube.
// Cube size is N^3 and M = N/2.

fftx_iodimx oct00[] = {
  { M+1, 0, 0, 0, 1, 1, 1 },
  { M+1, 0, 0, 0, M+1, 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} },
  oct01[] = {
  { M-1, M-1, M+1, 0, -1, 1, 1 },
  { M+1, 0, 0, 0, M+1, 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} },
  oct10[] = {
  { M+1, 0, 0, 0, 1, 1, 1 },
  { M-1, M-1, M+1, 0, -(M+1), 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} },
 oct11[] = {
  { M-1, M-1, M+1, 0, -1, 1, 1 },
  { M-1, M-1, M+1, 0, -(M+1), 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} };
...
```

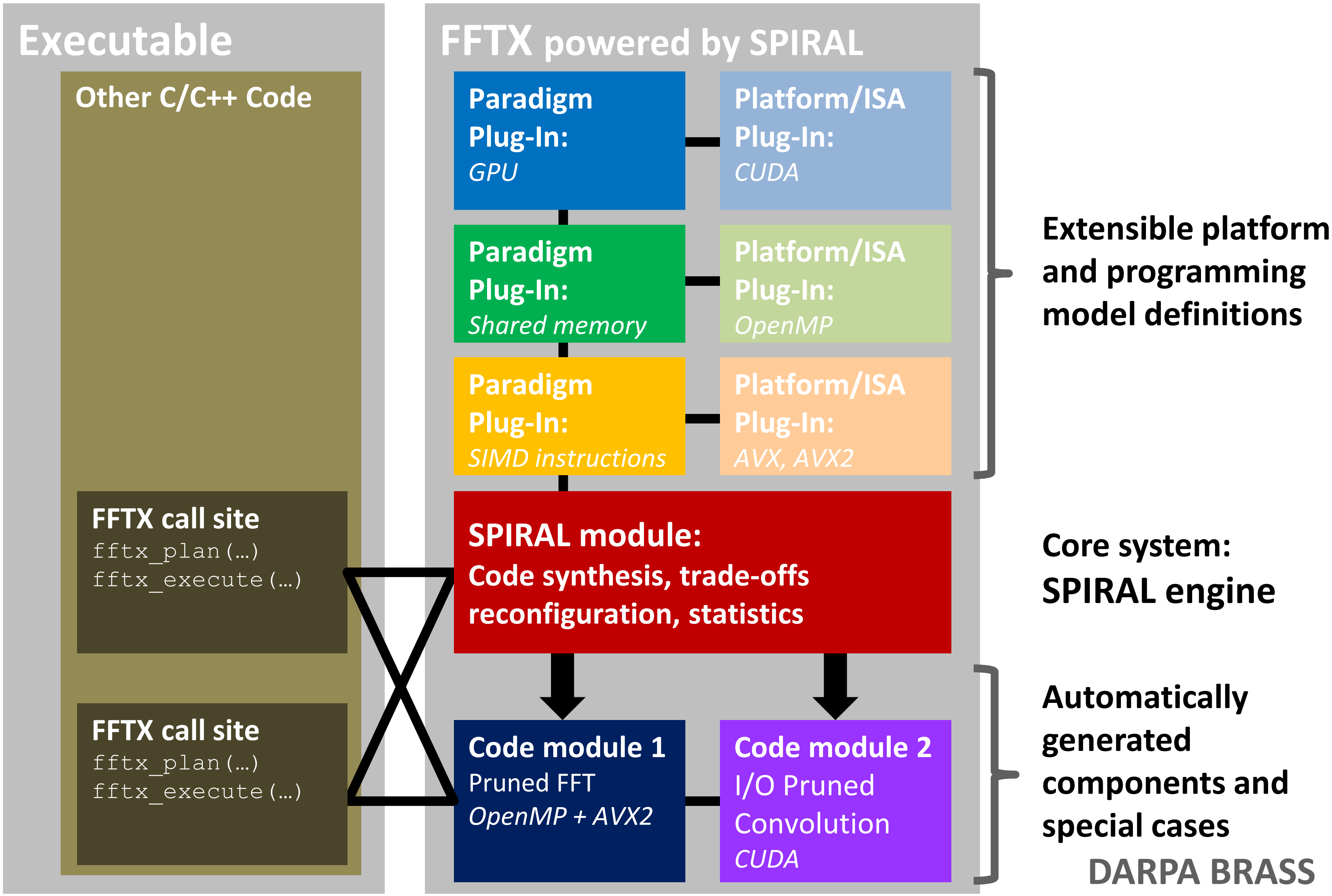


$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

```
fftx_temp_complex half_G_k = fftx_create_zero_temp_complex(rk, f_d);
plans[2] = fftx_plan_guru_copy_complex(rk, oct00, G_k, half_G_k, FFTX_MODE_SUB);
plans[3] = fftx_plan_guru_copy_complex(rk, oct01, G_k, half_G_k, MY_FFTX_MODE_SUB);
plans[4] = fftx_plan_guru_copy_complex(rk, oct10, G_k, half_G_k, MY_FFTX_MODE_SUB);
plans[5] = fftx_plan_guru_copy_complex(rk, oct11, G_k, half_G_k, MY_FFTX_MODE_SUB);
...
```

We are a developing higher-level more natural geometric API

FFTX Backend: SPIRAL



DARPA BRASS

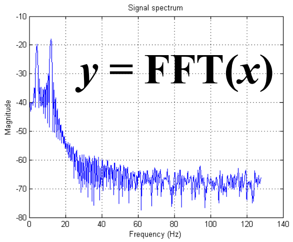
SPIRAL: Go from Mathematics to Software

Given:

- **Mathematical problem specification**
  core mathematics does not change
- **Target computer platform**
  varies greatly, new platforms introduced often

Wanted:

- **Very good implementation of specification on platform**
- **Proof of correctness**



on

automatic →

```
void fft64(double  *Y, double  *X) {
    ...
    s5674 = _mm256_permute2f128_pd(s5672, s5673, (0) | ((2) << 4));
    s5675 = _mm256_permute2f128_pd(s5672, s5673, (1) | ((3) << 4));
    s5676 = _mm256_unpacklo_pd(s5674, s5675);
    s5677 = _mm256_unpackhi_pd(s5674, s5675);
    s5678 = *((a3738 + 16));
    s5679 = *((a3738 + 17));
    s5680 = _mm256_permute2f128_pd(s5678, s5679, (0) | ((2) << 4));
    s5681 = _mm256_permute2f128_pd(s5678, s5679, (1) | ((3) << 4));
    s5682 = _mm256_unpacklo_pd(s5680, s5681);
    s5683 = _mm256_unpackhi_pd(s5680, s5681);
    t5735 = _mm256_add_pd(s5676, s5682);
    t5736 = _mm256_add_pd(s5677, s5683);
    t5737 = _mm256_add_pd(s5670, t5735);
    t5738 = _mm256_add_pd(s5671, t5736);
    t5739 = _mm256_sub_pd(s5670, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5735));
    t5740 = _mm256_sub_pd(s5671, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5736));
    t5741 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5677, s5683));
    t5742 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5676, s5682));
    ...
}
```

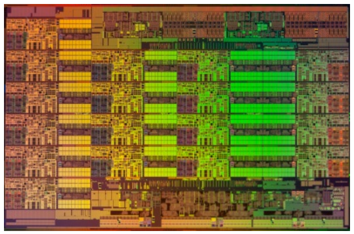
performance

+

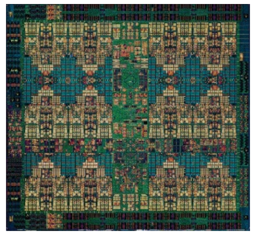
QED.

SPIRAL’s Target Computing Landscape

1 Gflop/s = one billion floating-point operations (additions or multiplications) per second

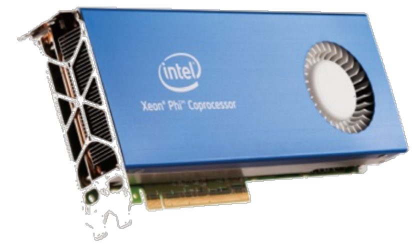


Intel Xeon 8180M
2.25 Tflop/s, 205 W
28 cores, 2.5—3.8 GHz
2-way—16-way AVX-512



IBM POWER9
768 Gflop/s, 300 W
24 cores, 4 GHz
4-way VSX-3

Nvidia Tesla V100
7.8 Tflop/s, 300 W
5120 cores, 1.2 GHz
32-way SIMT



Intel Xeon Phi 7290F
1.7 Tflop/s, 260 W
72 cores, 1.5 GHz
8-way/16-way LRBni

Snapdragon 835
15 Gflop/s, 2 W
8 cores, 2.3 GHz
A540 GPU, 682 DSP, NEON

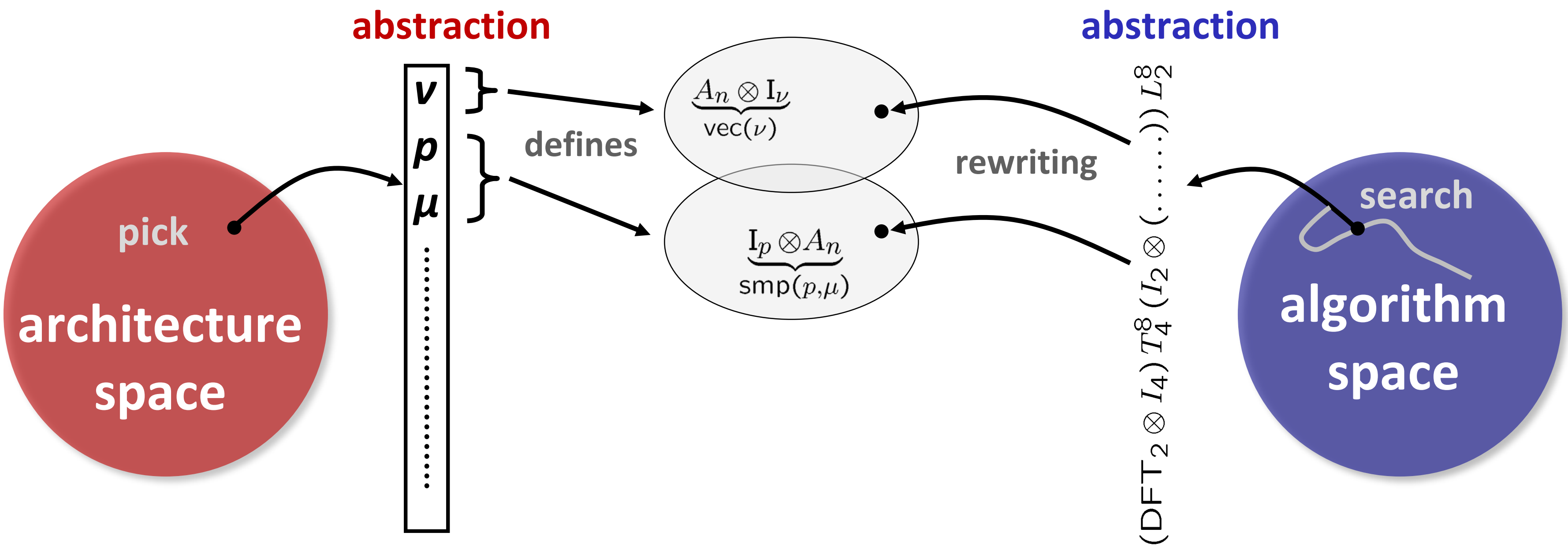
Intel Atom C3858
32 Gflop/s, 25 W
16 cores, 2.0 GHz
2-way/4-way SSSE3

Dell PowerEdge R940
3.2 Tflop/s, 6 TB, 850 W
4x 24 cores, 2.1 GHz
4-way/8-way AVX

Summit
187.7 Pflop/s, 13 MW
9,216 x 22 cores POWER9
+ 27,648 V100 GPUs

Platform-Aware Formal Program Synthesis

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

optimization

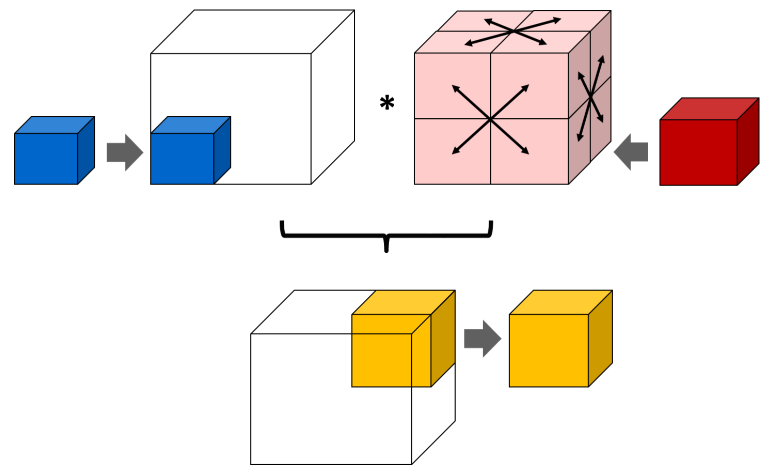
Kernel:
problem size,
algorithm choice

Rules in Internal Domain Specific Language

Linear Transforms

$$
\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n (\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \mathrm{gcd}(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\, \mathrm{L}^n_m (\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\qquad \cdot (\mathrm{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1} \\ & \frac{1}{\sqrt{2}} (\mathrm{I}_1 \oplus 2\, \mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1 + \cdots + k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1} + \cdots + k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathrm{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathrm{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathrm{J}_2\, \mathrm{R}_{13\pi/8}
\end{aligned}
$$

Spectral Domain Algorithms



Hardware

- **Multithreading (Multicore)** — $\mathrm{I}_p \otimes_{\|} A_{\mu n}, \quad \mathbf{L}^{mn}_m \bar{\otimes} \mathrm{I}_\mu$
- **Vector SIMD (SSE, VMX/Altivec,…)** — $A \hat{\otimes} \mathrm{I}_\nu \quad \underbrace{\mathbf{L}^{2\nu}_2}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{2\nu}_\nu}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{\nu^2}_\nu}_{\text{isa}}$
- **Message Passing (Clusters, MPP)** — $\mathrm{I}_p \otimes_{\|} A_n, \quad \underbrace{\mathbf{L}^{p^2}_p \bar{\otimes} \mathrm{I}_{n/p^2}}_{\text{all-to-all}}$
- **Streaming/multibuffering (Cell)** — $\mathrm{I}_n \otimes_2 A_{\mu n}, \quad \mathbf{L}^{mn}_m \bar{\otimes} \mathrm{I}_\mu$
- **Graphics Processors (GPUs)** — $\prod_{i=0}^{n-1} A_i, \quad A_n \hat{\otimes} \mathrm{I}_w, \quad P_n \otimes Q_w$
- **Gate-level parallelism (FPGA)** — $\prod_{i=0}^{n-1}{}^{\text{ir}} A, \quad \mathrm{I}_s \tilde{\otimes} A, \quad \underbrace{\mathbf{L}^m_n}_{\text{bram}}$
- **HW/SW partitioning (CPU + FPGA)** — $\underbrace{A_1}_{\text{fpga}}, \quad \underbrace{A_2}_{\text{fpga}}, \quad \underbrace{A_3}_{\text{fpga}}, \quad \underbrace{A_4}_{\text{fpga}}$

Program Transformations

$$
\begin{aligned}
\underbrace{AB}_{\text{smp}(p,\mu)} &\rightarrow \underbrace{A}_{\text{smp}(p,\mu)} \underbrace{B}_{\text{smp}(p,\mu)} \\
\underbrace{A_m \otimes \mathrm{I}_n}_{\text{smp}(p,\mu)} &\rightarrow \underbrace{\left(\mathrm{L}^{mp}_m \otimes \mathrm{I}_{n/p} \right) \left(\mathrm{I}_p \otimes (A_m \otimes \mathrm{I}_{n/p}) \right) \left(\mathrm{L}^{mp}_p \otimes \mathrm{I}_{n/p} \right)}_{\text{smp}(p,\mu)} \\
\underbrace{\mathrm{L}^{mn}_m}_{\text{smp}(p,\mu)} &\rightarrow \begin{cases} \underbrace{\left(\mathrm{I}_p \otimes \mathrm{L}^{mn/p}_{m/p} \right)}_{\text{smp}(p,\mu)} \underbrace{\left(\mathrm{L}^{pn}_p \otimes \mathrm{I}_{m/p} \right)}_{\text{smp}(p,\mu)} \\ \underbrace{\left(\mathrm{L}^{pm}_m \otimes \mathrm{I}_{n/p} \right)}_{\text{smp}(p,\mu)} \underbrace{\left(\mathrm{I}_p \otimes \mathrm{L}^{mn/p}_m \right)}_{\text{smp}(p,\mu)} \end{cases}
\end{aligned}
$$

Recursive rules

$$
\begin{aligned}
\underbrace{\mathrm{I}_m \otimes A_n}_{\text{smp}(p,\mu)} &\rightarrow \mathrm{I}_p \otimes_{\|} \left(\mathrm{I}_{m/p} \otimes A_n \right) \\
\underbrace{(P \otimes \mathrm{I}_n)}_{\text{smp}(p,\mu)} &\rightarrow \left(P \otimes \mathrm{I}_{n/\mu} \right) \bar{\otimes}\, \mathrm{I}_\mu
\end{aligned}
$$

Base case rules

Autotuning in Constraint Solution Space

$\underbrace{\mathrm{DFT}_8}_{\mathrm{AVX}(2\text{-way }\mathbb{C})}$

AVX 2-way `_Complex double`

DFT_8

Base cases

$A^{n\times n}\vec{\otimes}\mathrm{I}_2$

$\underbrace{\mathrm{L}^4_2}_{\mathrm{vec}(2)}$

$\underbrace{\mathrm{T}^{mn}_n}_{\mathrm{vec}(2)}$

Transformation rules

$$(\mathrm{I}_m\otimes A^{n\times n})\mathrm{L}^{mn}_m \to \left(\mathrm{I}_{m/\nu}\otimes \mathrm{L}^{n\nu}_\nu(A^{n\times n}\otimes \mathrm{I}_\nu)\right)(\mathrm{L}^{mn/\nu}_{m/\nu}\otimes \mathrm{I}_\nu)$$

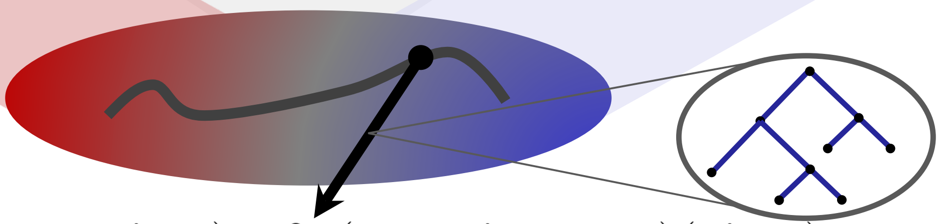
$$\mathrm{L}^{n\nu}_\nu \to (\mathrm{L}^n_\nu\otimes \mathrm{I}_\nu)(\mathrm{I}_{n/\nu}\otimes \mathrm{L}^{\nu^2}_\nu)$$

$$A^{m\times m}\otimes \mathrm{I}_n \to (A^{m\times m}\otimes \mathrm{I}_{n/\nu})\otimes \mathrm{I}_\nu$$

Breakdown rules

$$\mathrm{DFT}_{mn} \to (\mathrm{DFT}_m\otimes \mathrm{I}_n)\mathrm{T}^{mn}_n(\mathrm{I}_m\otimes \mathrm{DFT}_n)\mathrm{L}^{mn}_m$$

$$\mathrm{DFT}_2 \to \mathrm{F}_2$$



$$\left((\mathrm{F}_2\otimes \mathrm{I}_2)\mathrm{T}^4_2(\mathrm{I}_2\otimes \mathrm{F}_2)\mathrm{L}^4_2\vec{\otimes}\mathrm{I}_2\right)\underbrace{\mathrm{T}^8_2}_{\mathrm{vec}(2)}\left(\mathrm{I}_2\otimes \underbrace{\mathrm{L}^4_2}_{\mathrm{vec}(2)}(\mathrm{F}_2\vec{\otimes}\mathrm{I}_2)\right)\left(\mathrm{L}^4_2\vec{\otimes}\mathrm{I}_2\right)$$

FFTX C/C++ specification code

↓ Inspector/executor

OL specification

↓ **Expansion + backtracking**

OL (dataflow) expression

↓ Recursive descent

Σ-OL (loop) expression

↓ Confluent term rewriting

Optimized Σ-OL expression

↓ Recursive descent

Abstract code

↓ Confluent term rewriting

Optimized abstract code

↓ Recursive descent

C code module

Translating an OL Expression Into Code

Constraint Solver Input: $\underbrace{\mathrm{DFT}_8}_{\mathrm{AVX(2\text{-}way}\ \mathbb{C})}$

Output =

Ruletree, expanded into

OL Expression:

$$\Big(\big((\mathbf{F}_2\otimes\mathbf{I}_2)\mathbf{T}^4_2(\mathbf{I}_2\otimes\mathbf{F}_2)\mathbf{L}^4_2\vec{\otimes}\,\mathbf{I}_2\big)\ \underbrace{\mathbf{T}^8_2}_{\mathrm{vec}(2)}\ \big(\mathbf{I}_2\otimes\ \underbrace{\mathbf{L}^4_2}_{\mathrm{vec}(2)}\ (\mathbf{F}_2\vec{\otimes}\,\mathbf{I}_2)\big)\Big)\big(\mathbf{L}^4_2\vec{\otimes}\,\mathbf{I}_2\big)$$

∑-OL:

$$\Big(\sum_{j=0}^{1}\big(\mathbf{S}_{\imath_2\otimes(j)_2}\mathbf{F}_2\mathrm{Map}^2_{x\mapsto\omega_4^{2i+j}x}\mathbf{G}_{\imath_2\otimes(j)_2}\big)\sum_{j=0}^{1}\big(\mathbf{S}_{(j)_2\otimes\imath_2}\mathbf{F}_2\mathbf{G}_{\imath_2\otimes(j)_2}\big)\Big)\vec{\otimes}\,\mathbf{I}_2\ldots$$

C Code:

```
void dft8(_Complex double *Y, _Complex double *X) {
    __m256d s38, s39, s40, s41,...
    __m256d  *a17, *a18;
    a17 = ((__m256d  *) X);
    s38 = *(a17);
    s39 = *((a17 + 2));
    t38 = _mm256_add_pd(s38, s39);
    t39 = _mm256_sub_pd(s38, s39);
    ...
    s52 = _mm256_sub_pd(s45, s50);
    *((a18 + 3)) = s52;
}
```

FFTX C/C++ specification code

↓ Inspector/executor

OL specification

↓ Expansion + backtracking

OL (dataflow) expression

↓ Recursive descent

Σ-OL (loop) expression

↓ Confluent term rewriting

Optimized Σ-OL expression

↓ Recursive descent

Abstract code

↓ Confluent term rewriting

Optimized abstract code (icode)

↓ Recursive descent

C code module

Generated Code For Hockney Convolution

```
void ioprunedconv_130_0_62_72_130(double  *Y, double  *X, double * S) {
    static double D84[260] = {65.5, 0.0, (-0.50000000000001132), (-20.686114762237267),
      (-0.50000000000000081), (-10.337014680426078), (-0.50000000000000455),
    ...
for(int i18899 = 0; i18899 <= 1; i18899++) {
    for(int i18912 = 0; i18912 <= 4; i18912++) {
        a9807 = ((2*i18899) + (4*i18912));
        a9808 = (a9807 + 1);
        a9809 = (a9807 + 52);
        a9810 = (a9807 + 53);
        a9811 = (a9807 + 104);
        a9812 = (a9807 + 105);
        s3295 = (*((X + a9807)) + *((X + a9809))
            + *((X + a9811)));
        s3296 = (*((X + a9808)) + *((X + a9810))
            + *((X + a9812)));
        s3297 = (((0.3090169943749474**((X + a9809))
            - (0.80901699437494745**((X + a9811))))
            + *((X + a9807)));
        ...
        *((104 + Y + a12569)) = ((s3983 - s3987)
            + (0.80901699437494745*t6537)
            + (0.58778525229247314*t6538));
        *((105 + Y + a12569)) = (((s3984 - s3988)
            + (0.80901699437494745*t6538))
            - (0.58778525229247314*t6537));
    }
}
```

FFTX/SPIRAL with OpenACC backend

Compared to cuFFT expert interface

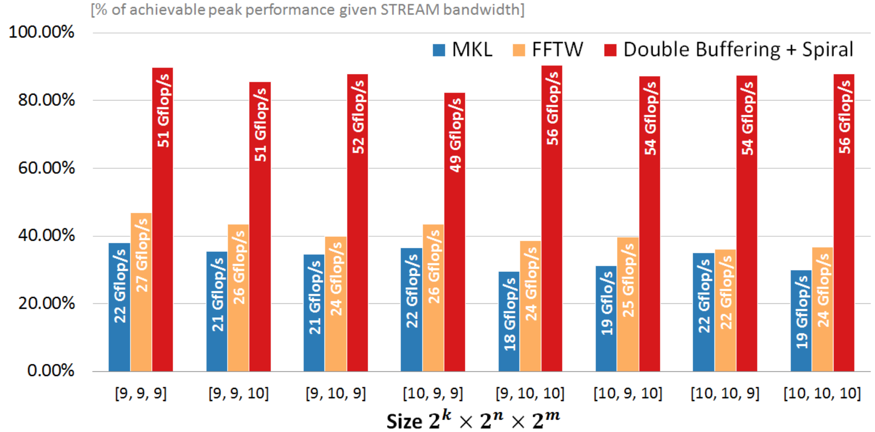
15% faster on TITAN V

Same speed on Tesla V100

1,000s of lines of code, cross call optimization, etc., transparently used

Selected Results: FFTs and Spectral Algorithms

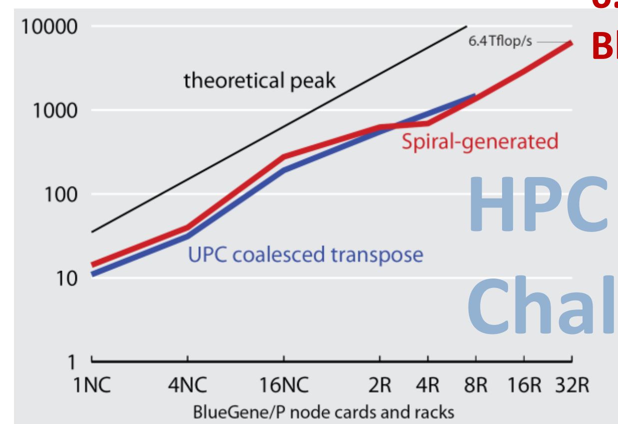
3D FFT performance on Intel Kaby Lake 7700K
4.5 GHz, 4/8 cores/threads, double-precision, AVX
[% of achievable peak performance given STREAM bandwidth]



FFT on Multicore

Global FFT (1D FFT, HPC Challenge)
performance [Gflop/s]

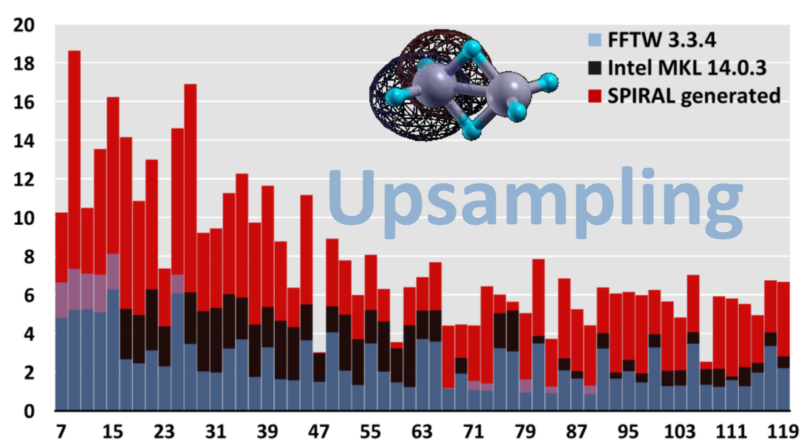
6.4 Tflop/s on BlueGene/P



HPC Challenge

BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz

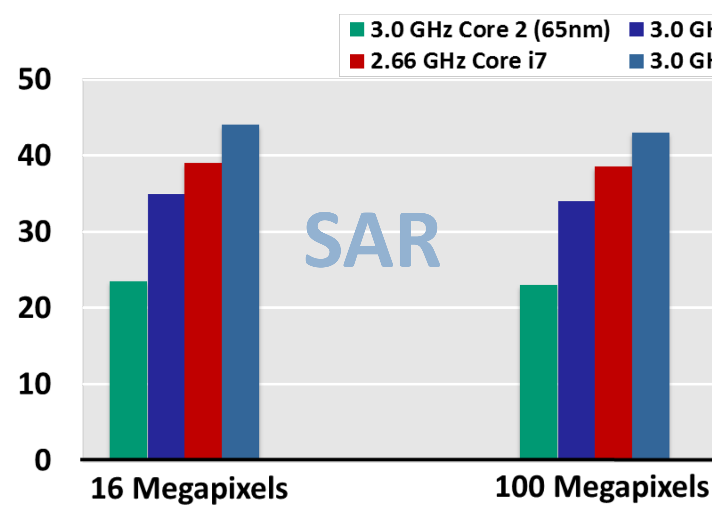
Performance of 2x2x2 Upsampling on Haswell
3.5 GHz, AVX, double precision, interleaved input, single core
Performance [Pseudo Gflop/s]



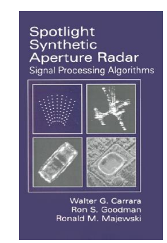
Upsampling

Input data cube edge length

PFA SAR Image Formation on Intel platforms
performance [Gflop/s]

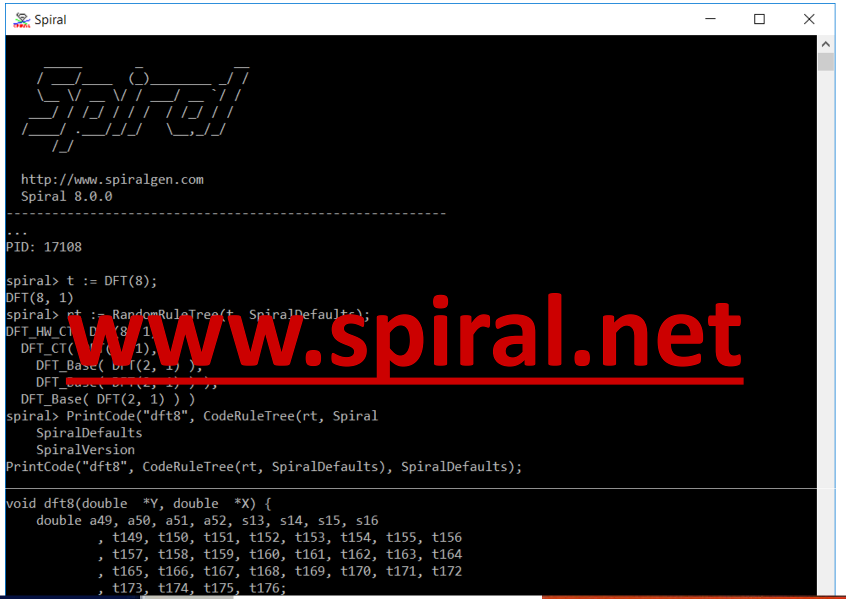


SAR



FFTX and SPIRAL 8.0: Open Source

- **Open Source SPIRAL available**
  - non-viral license (BSD)
  - Initial version, effort ongoing to open source whole system
  - Open sourced under DARPA PERFECT
  - Commercial support via SpiralGen, Inc.



- **Developed over 20 years**
  Funding: DARPA (OPAL, DESA, HACMS, PERFECT, BRASS), NSF, ONR, DoD HPC, JPL, DOE, CMU SEI, Intel, Nvidia, Mercury

- **FFTX 1.0 announced for late 2019**
  http://www.spiral.net/docs/fftx

- **SPIRAL and FFTX Tutorial**
  planned at IEEE HPEC 2019
  http://www.ieee-hpec.org

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Special Issue on *From High Level Specification to High Performance Code*