Formal Loop Merging for Signal Transforms

Franz Franchetti

Yevgen S. Voronenko

Markus Püschel

Department of Electrical & Computer Engineering

Carnegie Mellon University

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http://www.spiral.net

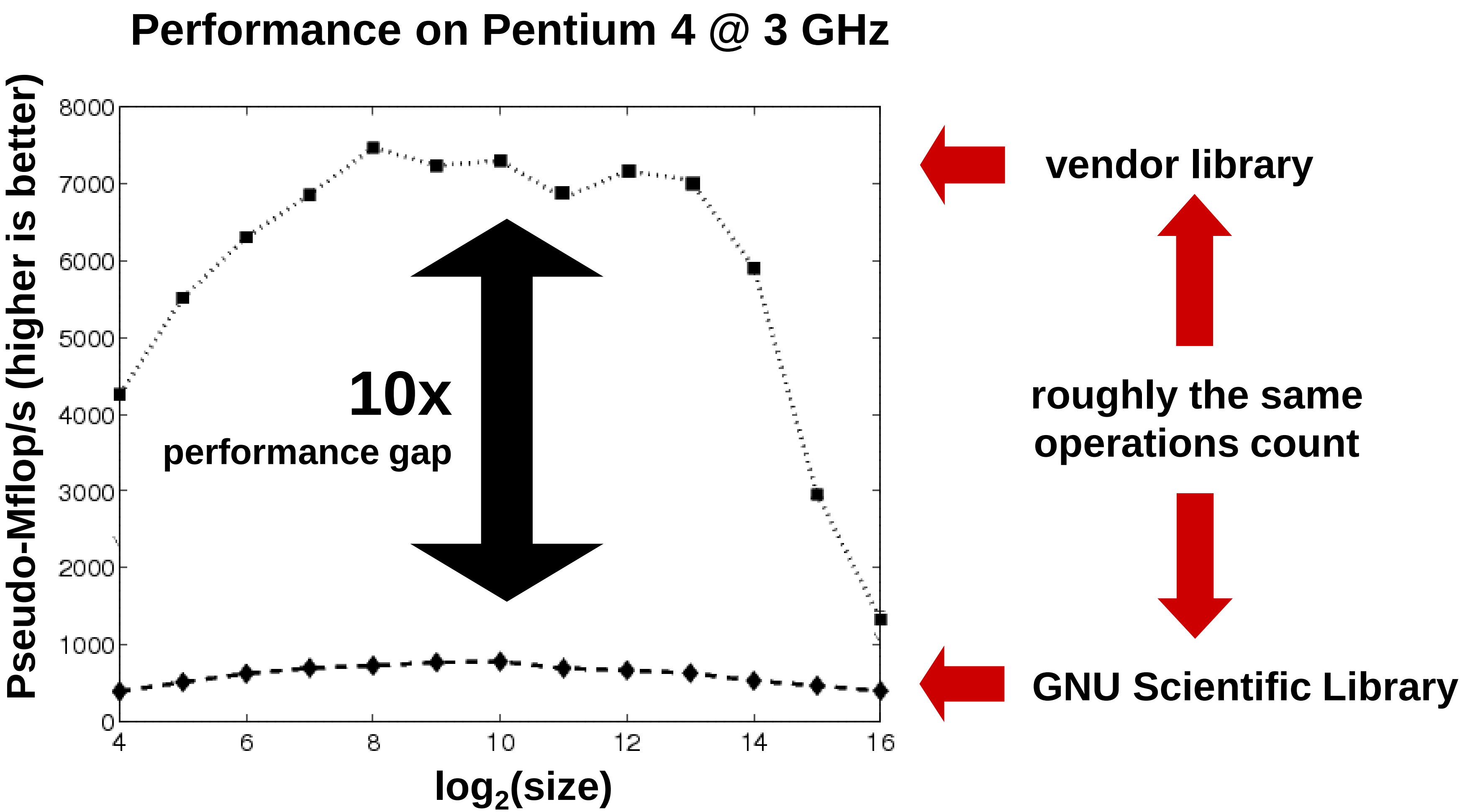
Problem

- **Runtime of (uniprocessor) numerical applications typically dominated by few compute-intensive kernels**
  - Examples: discrete Fourier transform, matrix-matrix multiplication

- **These kernels are hand-written for every architecture (open-source and commercial libraries)**

- **Writing fast numerical code is becoming increasingly difficult, expensive, and platform dependent, due to:**
  - Complicated memory hierarchies
  - Special purpose instructions (short vector extensions, fused multiply-add)
  - Other microarchitectural features (deep pipelines, superscalar execution)

Example: Discrete Fourier Transform (DFT)



Writing fast code is hard. Are there alternatives?

Automatic Code Generation and Adaptation

- **ATLAS:** Code generator for basic linear algebra subroutines (BLAS)
  [Whaley, et. al., 1998] [Yotov, et al., 2005]

- **FFTW:** Adaptive library for computing the discrete Fourier transform (DFT) and its variants
  [Frigo and Johnson, 1998]

- **SPIRAL:** Code generator for linear signal transforms (including DFT)
  [Püschel, et al., 2004]

- **See also:** Proceedings of the IEEE special issue on “Program Generation, Optimization, and Adaptation,” Feb. 2005.

- **Focus of this talk:**
  A new approach to automatic loop merging in SPIRAL

Talk Organization

- SPIRAL Background
- Automatic loop merging in SPIRAL
- Experimental Results
- Conclusions

Talk Organization

- **SPIRAL Background**
- **Automatic loop merging in SPIRAL**
- **Experimental Results**
- **Conclusions**

SPIRAL: DSP Transforms

- **SPIRAL generates optimized code for linear signal transforms, such as discrete Fourier transform (DFT), discrete cosine transforms, FIR filters, wavelets, and many others.**

- **Linear transform = matrix-vector product:**

$$y = Mx$$

output   transform matrix   input

- **Example: DFT of input vector X**

$$y = \mathrm{DFT}_n\, x$$

$$\mathrm{DFT}_n = \left[\omega_n^{k\ell}\right]_{0\le k,\ell<n}, \qquad \omega_n = e^{-2\pi\sqrt{-1}/n}$$

SPIRAL: Fast Transform Algorithms

- **Reduce computation cost from $O(n^2)$ to $O(n \log n)$**
- **For every transform there are many fast algorithms**
- **Algorithm = sparse matrix factorization**

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix} x \longrightarrow \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} x$$

12 adds
4 mults

4 adds **1 mult** **4 adds**

(when multiplied with input vector x)

$$\mathrm{DFT}_4 \rightarrow (\mathrm{DFT}_2 \otimes \mathrm{I}_2) D (\mathrm{I}_2 \otimes \mathrm{DFT}_2) P$$

- **SPIRAL generates the space of algorithms using breakdown rules in the domain-specific Signal Processing Language (SPL)**

$$\mathrm{DFT}_{mn} \rightarrow (\mathrm{DFT}_m \otimes \mathrm{I}_n) D (\mathrm{I}_m \otimes \mathrm{DFT}_n) P$$

SPIRAL

SPL (Signal Processing Language)

- **SPL expresses transform algorithms as structured sparse matrix factorization**

- **Examples:**

$$\mathsf{F}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad A \otimes B = \left[a_{k,\ell} B\right]_{k,\ell}$$

$$A \oplus B = \begin{bmatrix} A & \\ & B \end{bmatrix} \qquad \mathrm{I} \otimes B = \begin{bmatrix} B & & \\ & \ddots & \\ & & B \end{bmatrix}$$

- **SPL grammar in Backus-Naur form**

$$\begin{array}{rcl} \langle \text{spl} \rangle & ::= & \langle \text{generic} \rangle \mid \langle \text{symbol} \rangle \mid \langle \text{transform} \rangle \mid \\ & & \langle \text{spl} \rangle \cdot \cdots \cdot \langle \text{spl} \rangle \mid \\ & & \langle \text{spl} \rangle \oplus \ldots \oplus \langle \text{spl} \rangle \mid \\ & & \langle \text{spl} \rangle \otimes \cdots \cdot \otimes \langle \text{spl} \rangle \mid \\ & & \ldots \\ \langle \text{generic} \rangle & ::= & \mathrm{diag}(a_0, \ldots, a_{n-1}) \mid \ldots \\ \langle \text{symbol} \rangle & ::= & \mathrm{I}_n \mid \mathrm{J}_n \mid \mathrm{L}_k^n \mid \mathrm{R}_\alpha \mid \mathrm{F}_2 \mid \ldots \\ \langle \text{transform} \rangle & ::= & \mathbf{DFT}_n \mid \mathbf{WHT}_n \mid \mathbf{DCT\text{-}2}_n \mid \mathbf{Filt}_n\,(h[z]) \mid \ldots \end{array}$$

Compiling SPL to Code Using Templates

$$y = \mathsf{L}_n^{mn}\, x$$

```
for i=0..n-1
   for j=0..m-1
      y[i+n*j]=x[m*i+j]
```

$$y = (A_n \oplus B_m)x$$

```
y[0:1:n-1] = call A(x[0:1:n-1])
y[n:1:n+m-1] = call B(x[n:1:n+m-1])
```

$$y = (\mathrm{I}_n \otimes B_m)x$$

```
for i=0..n-1
   y[im:1:im+m-1] = call B(x[im:1:im+m-1])
```

$$y = (\mathrm{I}_n \otimes B_m)\, \mathsf{L}_n^{mn}\, x$$

```
for i=0..n-1
   y[im:1:im+m-1] = call B(x[i:n:i+m-1])
```

Some Transforms and Breakdown Rules in SPIRAL

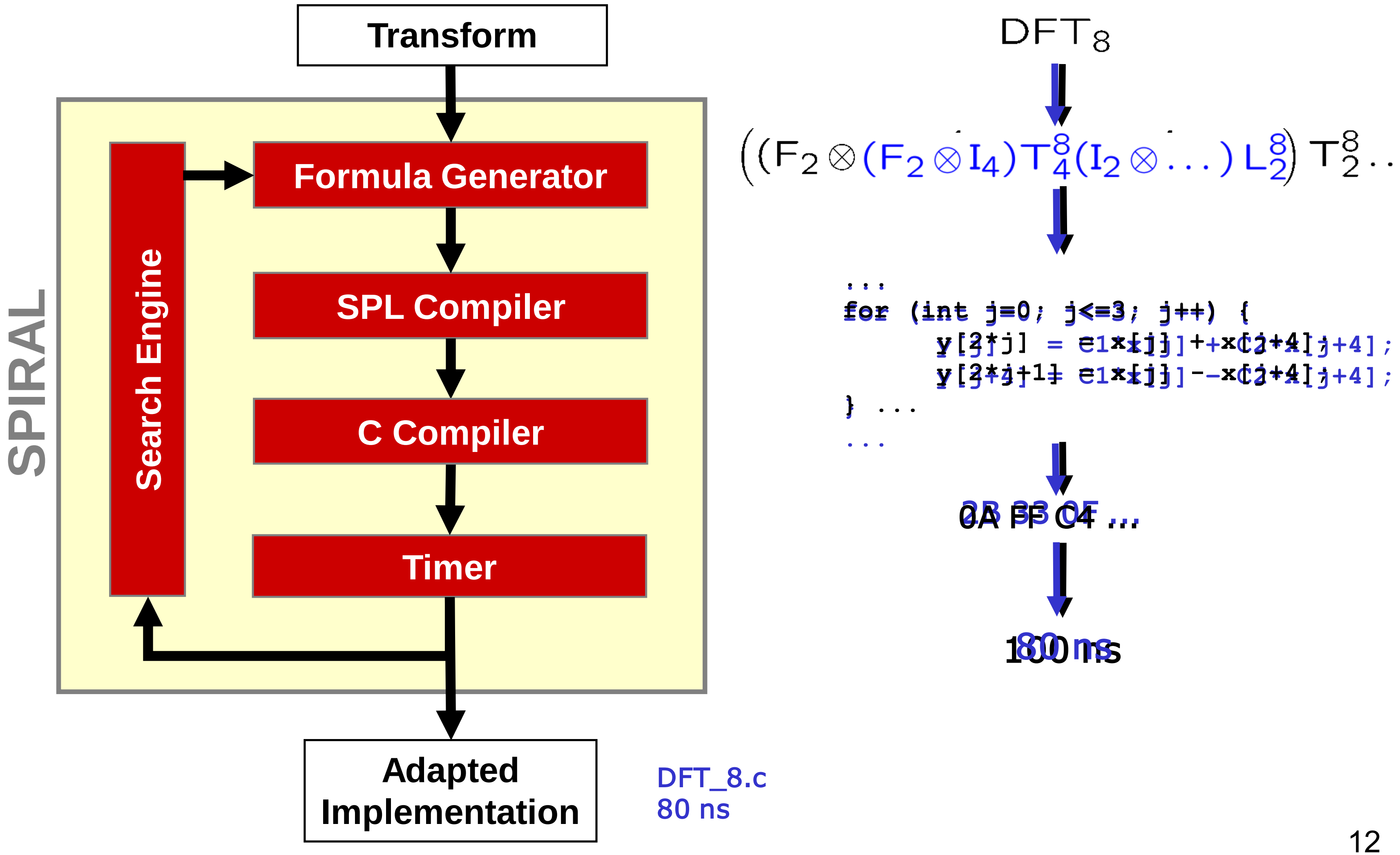
$$
\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathsf{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathsf{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \mathrm{gcd}(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathsf{J}_m)\, \mathsf{L}^n_m (\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\qquad \cdot (\mathsf{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathsf{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathsf{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathsf{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \right) \mathsf{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathsf{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathsf{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathsf{J}_2\, \mathsf{R}_{13\pi/8}
\end{aligned}
$$

Base case rules

Spiral contains 30+ transforms and 100+ rules

SPIRAL Architecture

Approach: Empirical search over alternative recursive algorithms



Problem: Fusing Permutations and Loops

$$y = (I_4 \otimes F_2)\, L^8_4\, x$$

Two passes over the working set
Complex index computation

direct mapping

```
void sub(double *y, double *x) {
   double t[8];
   for (int i=0; i<=7; i++)
      t[(i/4)+2*(i%4)] = x[i];
   for (int i=0; i<4; i++){
      y[2*i] = t[2*i] + t[2*i+1];
      y[2*i+1] = t[2*i] - t[2*i+1];
   }
}
```

C compiler cannot do this

hardcoded special case

One pass over the working set
Simple index computation

```
void sub(double *y, double *x) {
   for (int j=0; j<=3; j++){
      y[2*j] = x[j] + x[j+4];
      y[2*j+1] = x[j] - x[j+4];
   }
}
```

State-of-the-art
SPIRAL: Hardcoded with templates
FFTW: Hardcoded in the infrastructure

How does hardcoding scale?

General Loop Merging Problem

= permutations

$$
\begin{aligned}
\mathbf{DFT}_n &\to (\mathbf{DFT}_k \otimes \mathrm{I}_m)\,\mathsf{T}^n_m \boxed{(\mathrm{I}_k \otimes \mathbf{DFT}_m)\,\mathsf{L}^n_k}, && n = km\\
\mathbf{DFT}_n &\to P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m)Q_n, && n = km,\ \gcd(k,m) = 1\\
\mathbf{DFT}_p &\to R_p^T(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})D_p(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})R_p, && p \text{ prime}\\
\mathbf{DCT\text{-}3}_n &\to (\mathrm{I}_m \oplus \mathsf{J}_m)\mathsf{L}^n_m(\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4))\\
&\quad \cdot(\mathsf{F}_2 \otimes \mathrm{I}_m)\begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathsf{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, && n = 2m\\
\mathbf{DCT\text{-}4}_n &\to S_n\mathbf{DCT\text{-}2}_n\,\mathrm{diag}_{0\le k<n}(1/(2\cos((2k+1)\pi/4n)))\\
\mathbf{IMDCT}_{2m} &\to (\mathsf{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathsf{J}_m)\left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right)\right)\mathsf{J}_{2m}\,\mathbf{DCT\text{-}4}_{2m}
\end{aligned}
$$

- Combinatorial explosion: Implementing templates for all rules and all recursive combinations is unfeasible
- In many cases even theoretically not understood

Our Solution in SPIRAL

- **Loop merging at C code level: impractical**
- **Loop merging at SPL level: not possible**
- **Solution:**
  - **New language Σ-SPL – an abstraction level between SPL and code**
  - **Loop merging through Σ-SPL formula manipulation**

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- **Automatic loop merging in SPIRAL**
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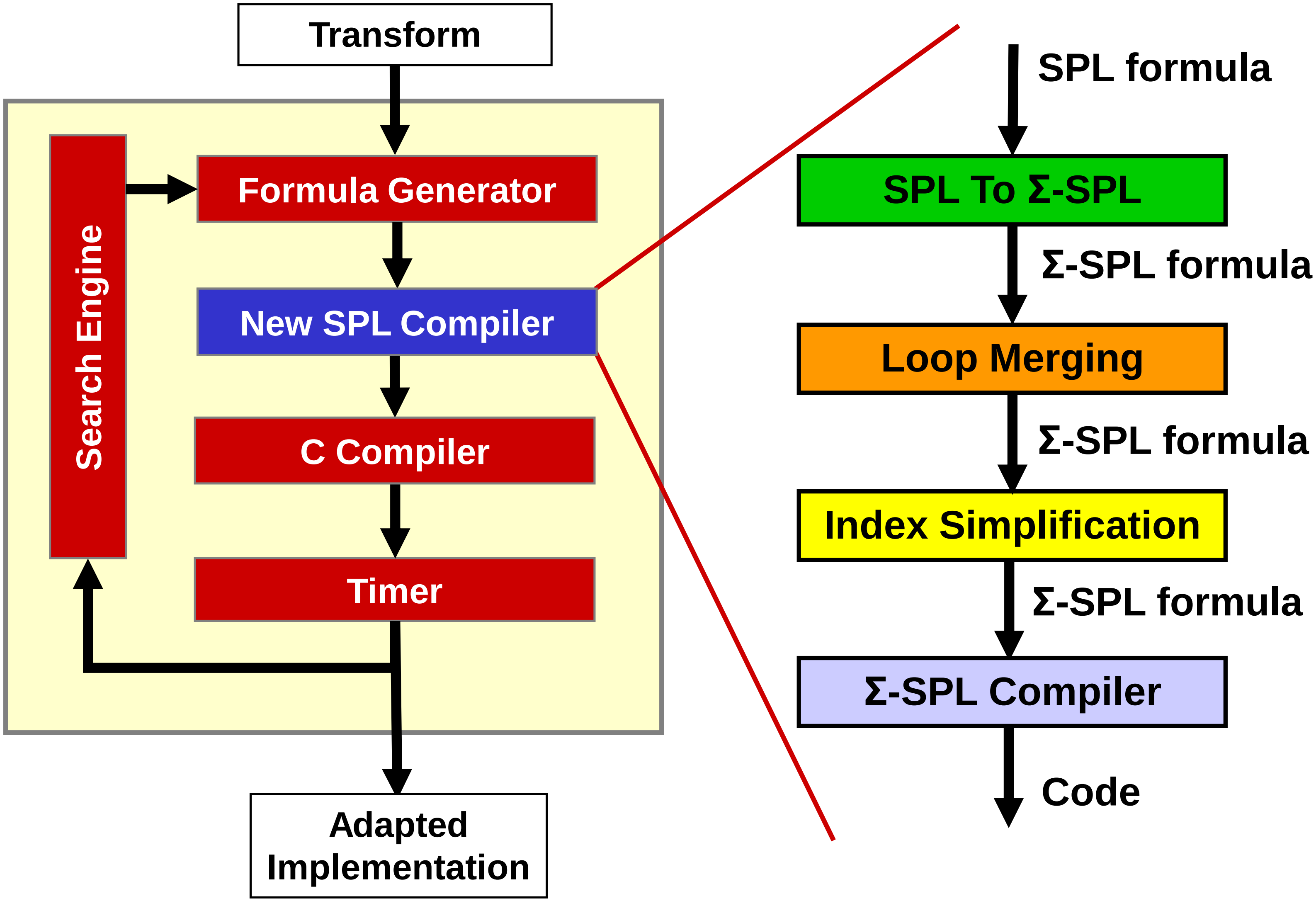
New Approach for Loop Merging



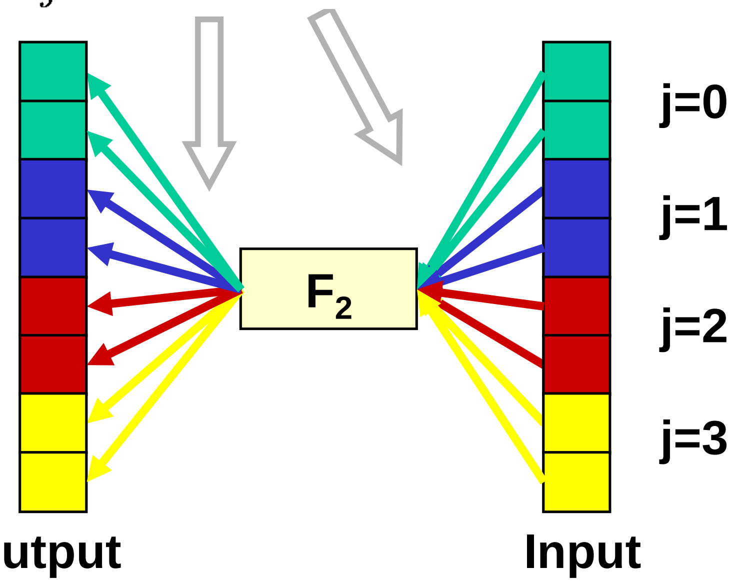
Σ–SPL

- Four central constructs: **Σ, G, S, Perm**
  - **Σ (sum)** – makes loops explicit
  - **G_f (gather)** – reads data using the index mapping f
  - **S_f (scatter)** – writes data using the index mapping f
  - **Perm_f** – permutes data using the index mapping f

- Every Σ-SPL formula still represents a matrix factorization

Example: $(I_4 \otimes F_2) \rightarrow \sum_{j=0}^{3} S_{f_j} F_2 G_{f_j}$

$$\begin{bmatrix} F_2 & & & \\ & F_2 & & \\ & & F_2 & \\ & & & F_2 \end{bmatrix}$$



Loop Merging With Rewriting Rules

SPL

SPL To Σ-SPL

Σ-SPL

Loop Merging

Σ-SPL

Index Simplification

Σ-SPL

Σ-SPL Compiler

Code

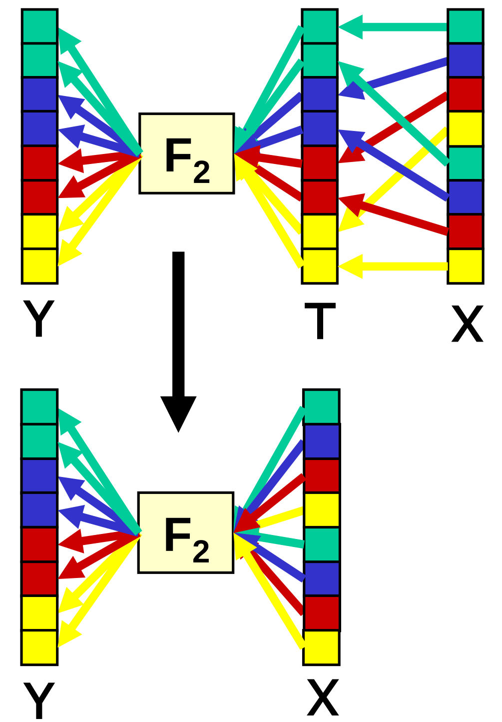
$$y = (\mathrm{I}_4 \otimes \mathrm{F}_2)\, \mathrm{L}^8_4\, x$$

$$\left(\sum_{j=0}^{3} \mathrm{S}_{(\jmath)_4 \otimes \imath_2}\, \mathrm{F}_2\, \mathrm{G}_{(\jmath)_4 \otimes \imath_2}\right) \mathrm{Perm}_{\ell^8_4}$$

$$\left(\sum_{j=0}^{3} \mathrm{S}_{(\jmath)_4 \otimes \imath_2}\, \mathrm{F}_2\, \mathrm{G}_{\ell^8_4 \circ ((\jmath)_4 \otimes \imath_2)}\right)$$

$$\left(\sum_{j=0}^{3} \mathrm{S}_{(\jmath)_4 \otimes \imath_2}\, \mathrm{F}_2\, \mathrm{G}_{\imath_2 \otimes (\jmath)_4}\right)$$

```
for (int j=0; j<=3; j++) {
      y[2*j]   = x[j] + x[j+4];
      y[2*j+1] = x[j] - x[j+4];
}
```



Rules:

$$\mathrm{G}_r\, \mathrm{Perm}_p \;\rightarrow\; \mathrm{G}_{p \circ r}$$

$$\ell^{mn}_m \circ \left((\jmath)_m \otimes \imath_n\right) \;\rightarrow\; \imath_n \otimes (\jmath)_m$$

Application: Loop Merging For FFTs

DFT breakdown rules:

Cooley-Tukey FFT $\mathbf{DFT}_{km} \rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathsf{T}^{km}_m (\mathrm{I}_k \otimes \mathbf{DFT}_m) \mathsf{L}^{km}_k$

Prime factor FFT $\mathbf{DFT}_{km} \rightarrow \mathsf{V}^T_{k,m} (\mathbf{DFT}_k \otimes \mathrm{I}_m)(\mathrm{I}_k \otimes \mathbf{DFT}_m) \mathsf{V}_{k,m}$

$\mathsf{gcd}(k,m) = 1$

Rader FFT $\mathbf{DFT}_p \rightarrow \mathsf{W}^T_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) \mathsf{W}_p$

p - prime

Index mapping functions are non-trivial:

$$\mathsf{L}^{km}_k \rightarrow \mathsf{Perm}_{\ell^{km}_k} \qquad \ell^{km}_k(i) = \left\lfloor \frac{i}{m} \right\rfloor + k(i \mod m)$$

$$\mathsf{V}_{k,m} \rightarrow \mathsf{Perm}_{v_{k,m}} \qquad v_{k,m}(i) = \left(m \left\lfloor \frac{i}{m} \right\rfloor + k(i \mod m)\right) \mod km$$

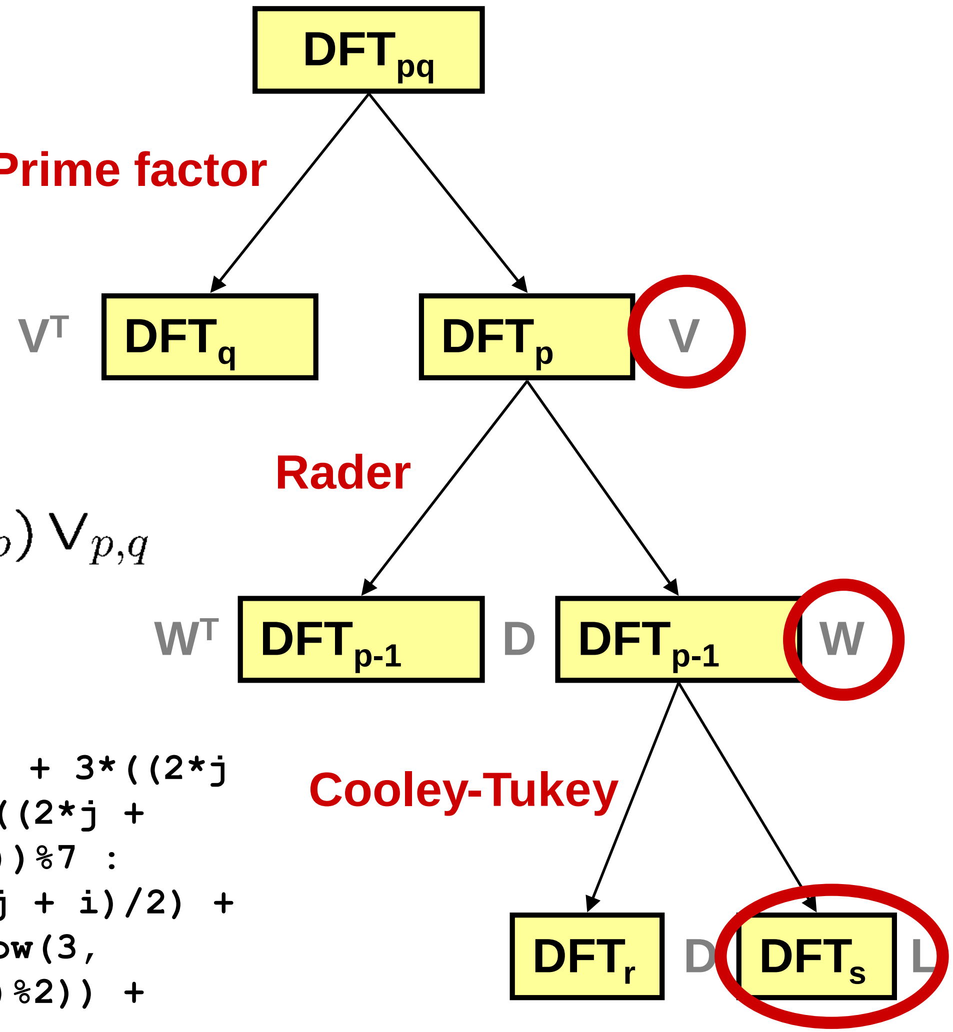
$$\mathsf{W}_p \rightarrow \mathsf{Perm}_{w^p_{1,g}} \qquad w^p_{\phi,g}(i) = \begin{cases} 0, & i = 0, \\ \phi g^i \mod p, & \text{else.} \end{cases}$$

Example

Given DFT_{pq}

p – prime

p-1 = rs



Formula fragment

$$(\mathrm{I}_p \otimes (\mathrm{I}_1 \oplus (\mathrm{I}_r \otimes \mathrm{DFT}_s)\, \mathrm{L}_r^{rs})\, \mathrm{W}_p)\, \mathrm{V}_{p,q}$$

Code for one memory access

```
p=7; q=4; r=3; s=2;
t=x[((21*((7*k + ((((((2*j + i)/2) + 3*((2*j
     + i)%2)) + 1)) ? (5*pow(3, ((((2*j +
     i)/2) + 3*((2*j + i)%2)) + 1)))%7 :
     (0)))/7) + 8*((7*k + ((((((2*j + i)/2) +
     3*((2*j + i)%2)) + 1)) ? (5*pow(3,
     ((((2*j + i)/2) + 3*((2*j + i)%2)) +
     1)))%7 : (0)))%7))%28)];
```

Task: Index simplification

Index Simplification: Basic Idea

- **Example: Identity necessary for fusing successive Rader and prime-factor step**

$$\left(\varphi g^{(b+si) \bmod N'}\right) \bmod N = \left((\varphi g^b)(g^s)^i\right) \bmod N$$

$$s|N',\ N'|N,\ 0 \le i < n$$

- **Performed at the Σ-SPL level through rewrite rules on function objects:**

$$\overline{w}_{\phi,g}^{N' \to N} \circ \overline{h}_{b,s}^{n \to N'} \quad \to \quad \overline{w}_{\phi g^b, g^s}^{n \to N}$$

- **Advantages:**
  - **no analysis necessary**
  - **efficient (or doable at all)**

Index Simplification Rules for FFTs

Cooley-Tukey

$$
\begin{aligned}
\ell_m^{mn} \circ \left((j)_m \otimes f^{k\to n} \right) &\to f^{k\to n} \otimes (j)_m \\
\left(f^{1\to m} \otimes h \right) \circ g &\to f \otimes (h \circ g) \\
\left(h \otimes g^{1\to n} \right) \circ f &\to (h \circ f) \otimes g \\
(f_0 \otimes f_1) \circ (g_0 \otimes g_1) &\to (f_0 \circ g_0) \otimes (f_1 \circ g_1)
\end{aligned}
$$

Transitional

$$
\begin{aligned}
\imath_n &\to h_{0,1}^{n\to n} \\
f^{m\to M} \otimes g^{1\to N} &\to h_{g(0),N}^{M\to MN} \circ f \\
g^{1\to N} \otimes f^{m\to M} &\to h_{Mg(0),1}^{M\to MN} \circ f
\end{aligned}
$$

Cooley-Tukey + Prime factor

$$
\begin{aligned}
v_{r,s} \circ h_{b,1}^{s\to rs} &\to \overline{h}_{b,r}^{s\to rs} \\
h_{b_1,s_1}^{nk\to mnk} \circ h_{b_2,s_2}^{n\to nk} &\to h_{b_1+s_1 b_2, s_1 s_2}^{n\to mnk} \\
\overline{h}_{b_1,s_1}^{nk\to mnk} \circ h_{b_2,s_2}^{n\to nk} &\to \overline{h}_{b_1+s_1 b_2, s_1 s_2}^{n\to mnk} \\
\overline{h}_{b_1,s_1}^{nk\to mnk} \circ \overline{h}_{b_2,s_2}^{n\to nk} &\to \overline{h}_{b_1+s_1 b_2, s_1 s_2}^{n\to mnk}
\end{aligned}
$$

Cooley-Tukey + Prime factor + Rader

$$
\begin{aligned}
w_{\phi,g}^{N} \circ (0)_{+}^{1\to N} &\to (0)_{+}^{1\to N} \\
w_{\phi,g}^{N} \circ (N-1)_{+}^{N-1\to N} &\to \overline{w}_{\phi,g}^{N-1\to N} \\
\overline{w}_{\phi,g}^{N'\to N} \circ h_{b,s}^{n\to N'} &\to \overline{w}_{\phi g^b, g^s}^{n\to N} \\
\overline{w}_{\phi,g}^{N'\to N} \circ \overline{h}_{b,s}^{n\to N'} &\to \overline{w}_{\phi g^b, g^s}^{n\to N}
\end{aligned}
$$

These 15 rules cover all combinations.
Some encode novel optimizations.

Loop Merging For the FFTs : Example (cont'd)

SPL

SPL To Σ-SPL

Σ-SPL

Loop Merging

Σ-SPL

Index Simplification

Σ-SPL

Σ-SPL Compiler

Code

$$(\mathrm{I}_p \otimes (\mathrm{I}_1 \oplus (\mathrm{I}_r \otimes \mathsf{DFT}_s)\, \mathsf{L}_r^{rs})\, \mathsf{W}_p)\, \mathsf{V}_{p,q}$$

$$\sum_{j_1=0}^{p-1} \Big(\mathsf{S}_{\left((j_1)_p \otimes \imath_q\right) \circ (0)_+^{1\to q}}\, \mathsf{G}_{v^{p,q} \circ \left((j_1)_p \otimes \imath_q\right) \circ w_{1,g}^{q} \circ (0)_+^{1\to q}}$$
$$+ \sum_{j_0=0}^{r-1} \mathsf{S}_{\left((j_1)_p \otimes \imath_q\right) \circ (1)_+^{q-1\to q} \circ ((j_0)_r \otimes \imath_s)}\, \mathsf{DFT}_s$$
$$\mathsf{G}_{v^{p,q} \circ \left((j_1)_p \otimes \imath_q\right) \circ w_{1,g}^{q} \circ (1)_+^{q-1\to q} \circ \ell_r^{rs} \circ ((j_0)_r \otimes \imath_s)} \Big)$$

$$\sum_{\substack{j_1=0 \\ b_1=qj_1}}^{p-1} \left(\mathsf{S}_{h_{0,q}^{p\to pq}(j_1)}\, \mathsf{G}_{\overline{h}_{0,q}^{p\to pq}(j_1)} + \sum_{\substack{j_0=0 \\ \phi_1=g^{j_0}}}^{r-1} \mathsf{S}_{h_{qj_1+sj_0+1,1}^{s\to pq}}\, \mathsf{DFT}_s\, \mathsf{G}_{\overline{h}_{b_1,p}^{q\to pq} \circ \overline{w}_{\phi_1,g^s}^{s\to q}} \right)$$

```
// I
in
fo
    y[7*j1] = x[(7*j1%28)];
    p1 = 1; b1 = 7*j1;
    for(int j0 = 0; j0 <= 2; j0++) {
        y[b1 + 2*j0 + 1] = x[(b1 + 4*p1)%28] + x[(b1 + 24*p1)%28];
        y[b1 + 2*j0 + 2] = x[(b1 + 4*p1)%28] - x[(b1 + 24*p1)%28];
        p1 = (p1*3%7);
    }
}
```

SPIRAL

```
// Input: _Complex double x[28], output: y[28]
double t1[28];
for(int i5 = 0; i5 <= 27; i5++)
    t1[i5] = x[(7*3*(i5/7) + 4*2*(i5%7))%28];
for(int i1 = 0; i1 <= 3; i1++) {
    double t3[7], t4[7], t5[7];
    for(int i6 = 0; i6 <= 6; i6++)
        t5[i6] = t1[7*i1 + i6];
    for(int i8 = 0; i8 <= 6; i8++)
        t4[i8] = t5[i8 ? (5*pow(3, i8))%7 : 0];
    {
        double t7[1], t8[1];
        t8[0] = t4[0];
        t7[0] = t8[0];
        t3[0] = t7[0];
    }
    {
        double t10[6], t11[6], t12[6];
        for(int i13 = 0; i13 <= 5; i13++)
            t12[i13] = t4[i13 + 1];
        for(int i14 = 0; i14 <= 5; i14++)
            t11[i14] = t12[(i14/2) + 3*(i14%2)];
        for(int i3 = 0; i3 <= 2; i3++) {
            double t14[2], t15[2];
            for(int i15 = 0; i15 <= 1; i15++)
                t15[i15] = t11[2*i3 + i15];
            t14[0] = (t15[0] + t15[1]);
            t14[1] = (t15[0] - t15[1]);
            for(int i17 = 0; i17 <= 1; i17++)
                 t10[2*i3 + i17] = t14[i17];
        }
        for(int i19 = 0; i19 <= 5; i19++)
            t3[i19 + 1] = t10[i19];
    }
    for(int i20 = 0; i20 <= 6; i20++)
        y[7*i1 + i20] = t3[i20];
}
```

```
// Input: _Complex double x[28], output: y[28]
  int p1, b1;
  for(int j1 = 0; j1 <= 3; j1++) {
     y[7*j1] = x[(7*j1%28)];
     p1 = 1; b1 = 7*j1;
     for(int j0 = 0; j0 <= 2; j0++) {
        y[b1 + 2*j0 + 1] = x[(b1 + 4*p1)%28] +
                           x[(b1 + 24*p1)%28];
        y[b1 + 2*j0 + 2] = x[(b1 + 4*p1)%28] -
                           x[(b1 + 24*p1)%28];
        p1 = (p1*3%7);
     }
  }
```

After, 2 Loops.

Before, 11 Loops.

SPIRAL

Talk Organization

- SPIRAL Background
- Automatic loop merging in SPIRAL
- **Experimental Results**
- Conclusions

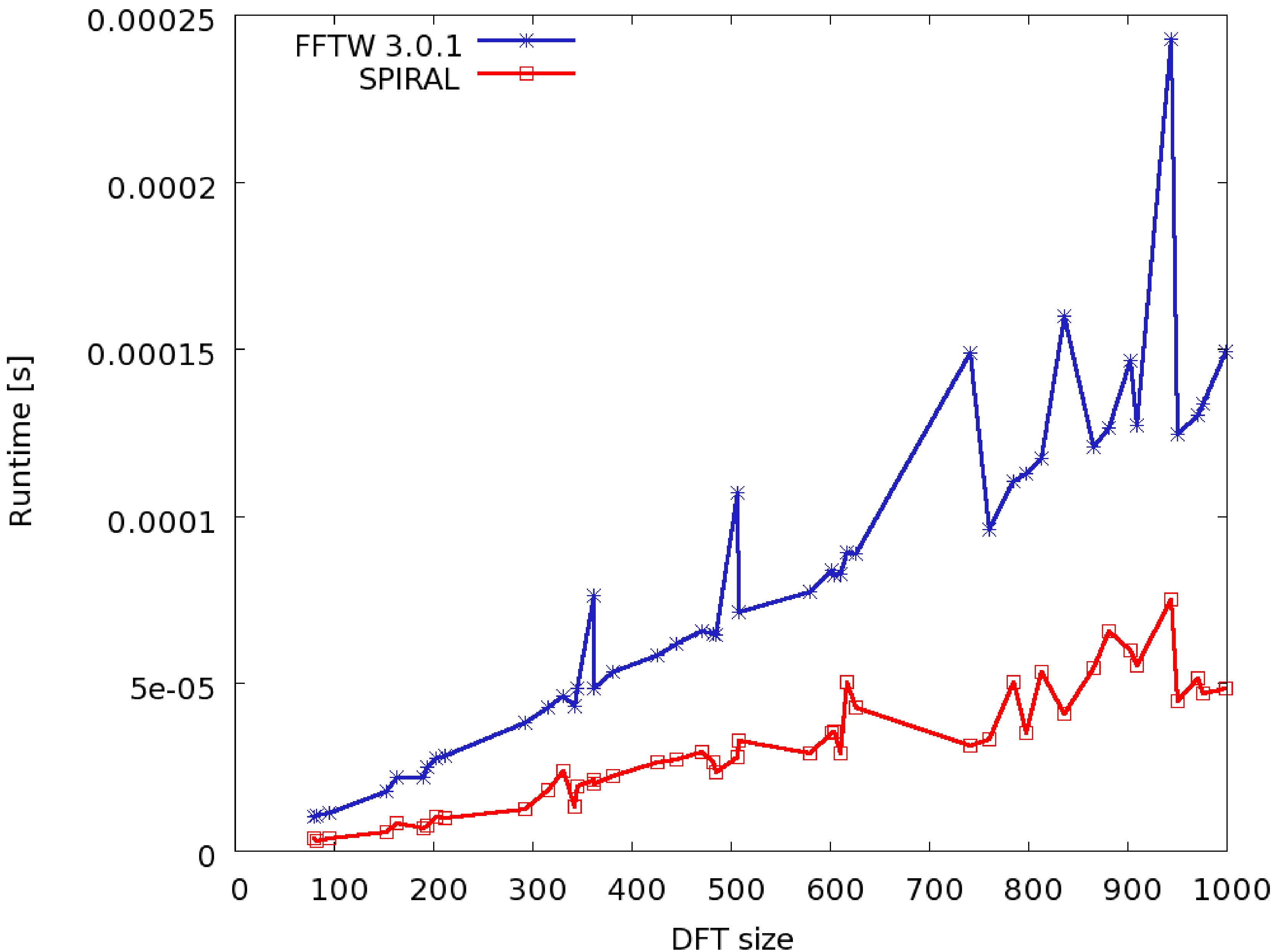
Benchmarks Setup

- Comparison against FFTW 3.0.1
- Pentium 4 3.6 GHz
- We consider sizes requiring at least one Rader step (sizes with large prime factor)
- We divide sizes into levels depending on number of Rader steps needed (Rader FFT has most expensive index mapping)

SPIRAL

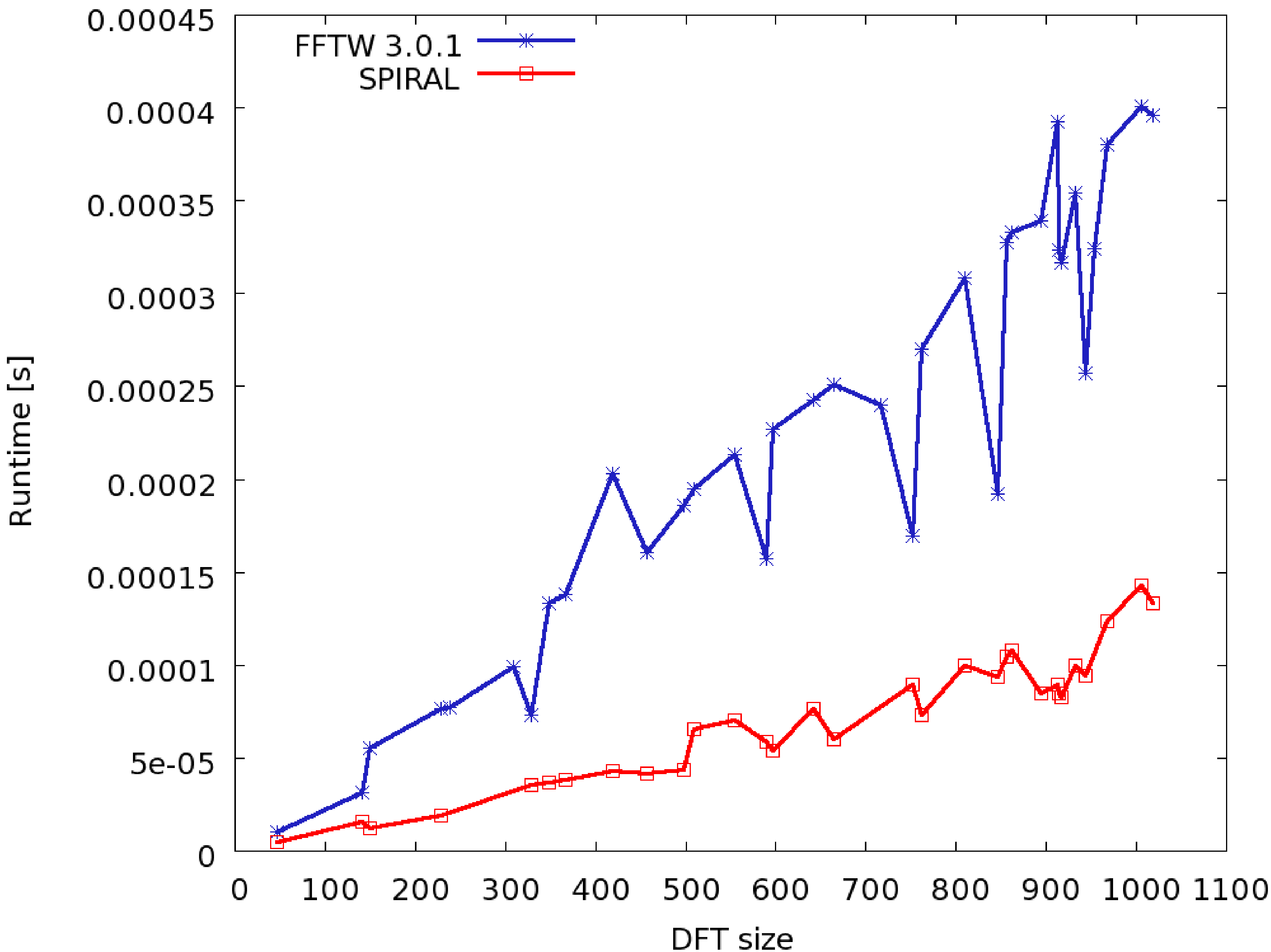
<u>One</u> Rader Step

Average SPIRAL speedup: <u>factor of 2.7</u>

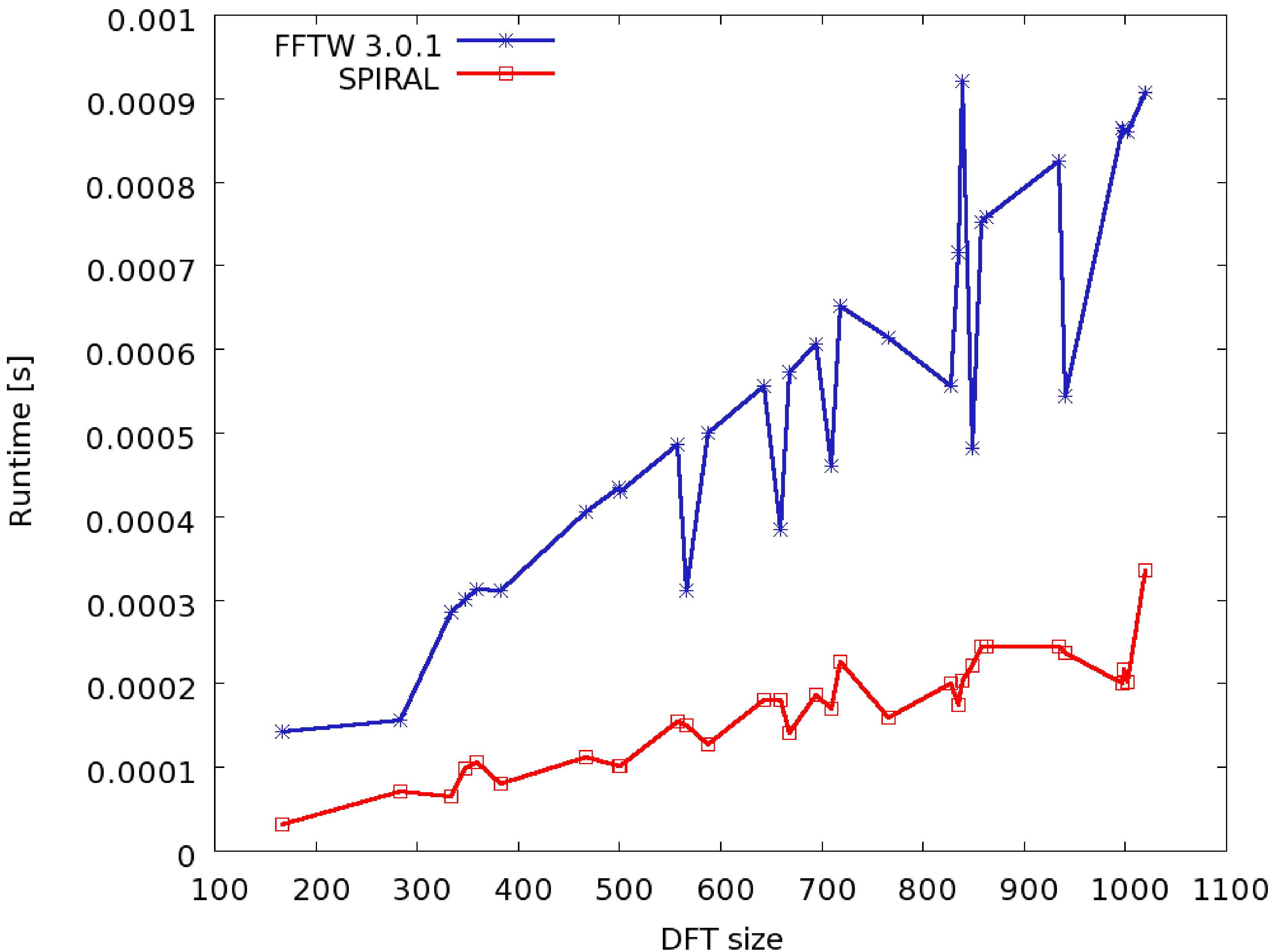


Two Rader Steps

Average SPIRAL speedup: factor of 3.3



Three Rader Steps **Average SPIRAL speedup: factor of 3.4**



Talk Organization

- SPIRAL Background
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Conclusion

- General loop optimization framework for linear DSP transforms in SPIRAL
- Loop optimization at the "right" abstraction level: Σ-SPL
- Application to FFT: Speedups of a factor of 2-5 over FFTW
- Future work: Other Σ-SPL optimizations
  - Loop merging for other transforms
  - Loop elimination, interchange, peeling

`http://www.spiral.net`

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