

Stride Permutations and Tensor Products

Franz Franchetti

August 4, 2006

1 Basic Identities

Identity 1.1 (Stride Identities).

$$\begin{aligned}L_n^{kmn} &= (L_n^{kn} \otimes I_m)(I_k \otimes L_n^{mn}) \\L_n^{kmn} &= L_{kn}^{kmn} L_{mn}^{kmn} \\L_{km}^{kmn} &= (I_k \otimes L_m^{mn})(L_k^{kn} \otimes I_m) \\L_{km}^{kmn} &= L_k^{kmn} L_m^{kmn}\end{aligned}$$

Identity 1.2 (Stride Factorizations).

$$\begin{aligned}(L_n^{kn} \otimes I_m)(I_k \otimes L_n^{mn}) &= L_n^{kmn} \\(L_k^{kn} \otimes I_m)L_n^{kmn} &= (I_k \otimes L_n^{mn}) \\L_n^{kmn}(I_k \otimes L_m^{mn}) &= (L_n^{kn} \otimes I_m) \\(I_k \otimes L_m^{mn})(L_k^{kn} \otimes I_m) &= L_{km}^{kmn} \\(I_k \otimes L_n^{mn})L_{km}^{kmn} &= (L_k^{kn} \otimes I_m) \\L_{km}^{kmn}(L_n^{kn} \otimes I_m) &= (I_k \otimes L_m^{mn})\end{aligned}$$

Identity 1.3 (Tensor Product Identities).

$$\begin{aligned}(I_m \otimes A^{n \times n}) &= L_m^{mn}(A^{n \times n} \otimes I_m)L_n^{mn} \\(A^{n \times n} \otimes I_m) &= L_n^{mn}(I_m \otimes A^{n \times n})L_m^{mn} \\(I_m \otimes A^{n \times n})L_m^{mn} &= L_m^{mn}(A^{n \times n} \otimes I_m) \\L_n^{mn}(I_m \otimes A^{n \times n}) &= (A^{n \times n} \otimes I_m)L_n^{mn}\end{aligned}$$

1.1 Parallel Identities

Identity 1.4 (Parallel Stride Identities).

$$\begin{aligned} L_n^{np} &= (I_p \otimes L_{n/p}^n)(L_p^{p^2} \otimes I_{n/p}) \\ L_p^{np} &= (L_p^{p^2} \otimes I_{n/p})(I_p \otimes L_p^n) \\ L_m^{mn} &= (I_p \otimes L_{m/p}^{mn/p})(L_p^{p^2} \otimes I_{mn/p^2})(I_p \otimes L_p^n \otimes I_{m/p}) \end{aligned}$$

Identity 1.5 (Parallel Leaf Identities).

$$\begin{aligned} (A^{m \times m} \otimes I_n) &= L_{mn/p}^{mn} (I_p \otimes (A^{m \times m} \otimes I_{n/p})) L_p^{mn} \\ (A^{m \times m} \otimes I_n) L_m^{mn} &= \left(L_m^{mp} (I_p \otimes A^{m \times m}) \otimes I_{n/p} \right) (I_p \otimes L_m^{mn/p}) \\ L_n^{mn} (A^{m \times m} \otimes I_n) &= (I_p \otimes L_{n/p}^{mn/p}) \left((I_p \otimes A^{m \times m}) L_p^{mp} \otimes I_{n/p} \right) \end{aligned}$$

Identity 1.6 (Parallel Recursion Identities).

$$\begin{aligned} \left((A^{k \times k} \otimes I_m) L_k^{km} \otimes I_n \right) L_{km}^{kmn} &= (I_k \otimes L_m^{mn}) \left((A^{k \times k} \otimes I_n) L_k^{kn} \otimes I_m \right) (I_n \otimes L_k^{km}) \\ L_n^{kmn} \left(L_m^{km} (A^{k \times k} \otimes I_m) \otimes I_n \right) &= (I_n \otimes L_m^{km}) \left(L_n^{kn} (A^{k \times k} \otimes I_n) \otimes I_m \right) (I_k \otimes L_n^{mn}) \\ (I_k \otimes A^{m \times m}) \otimes I_n &= (I_k \otimes L_m^{mn}) \left(I_k \otimes (I_n \otimes A^{m \times m}) \right) (I_k \otimes L_n^{mn}) \end{aligned}$$

1.2 Vector Identities

Identity 1.7 (Vector Stride Identities).

$$\begin{aligned} L_n^{n\nu} &= (I_{n/\nu} \otimes L_\nu^{\nu^2})(L_{n/\nu}^n \otimes I_\nu) \\ L_\nu^{n\nu} &= (L_\nu^n \otimes I_\nu)(I_{n/\nu} \otimes L_\nu^{\nu^2}) \\ L_m^{mn} &= (L_m^{mn/\nu} \otimes I_\nu)(I_{mn/\nu^2} \otimes L_\nu^{\nu^2})(I_{n/\nu} \otimes L_{m/\nu}^m \otimes I_\nu) \end{aligned}$$

Identity 1.8 (Vector Leaf Identities).

$$\begin{aligned} (I_m \otimes A^{n \times n}) &= L_\nu^{mn} ((I_{m/\nu} \otimes A^{n \times n}) \otimes I_\nu) L_{mn/\nu}^{mn} \\ (I_m \otimes A^{n \times n}) L_m^{mn} &= \left(I_{m/\nu} \otimes L_\nu^{n\nu} (A^{n \times n} \otimes I_\nu) \right) (L_{m/\nu}^{mn/\nu} \otimes I_\nu) \\ L_n^{mn} (I_m \otimes A^{n \times n}) &= (L_n^{mn/\nu} \otimes I_\nu) \left(I_{m/\nu} \otimes (A^{n \times n} \otimes I_\nu) L_n^{n\nu} \right) \end{aligned}$$

Identity 1.9 (Vector Recursion Identities).

$$\begin{aligned} \left(I_k \otimes (I_m \otimes A^{n \times n}) L_m^{mn} \right) L_k^{kmn} &= (L_k^{km} \otimes I_n) \left(I_m \otimes (I_k \otimes A^{n \times n}) L_k^{kn} \right) (L_m^{mn} \otimes I_k) \\ L_{mn}^{kmn} \left(I_k \otimes L_n^{mn} (I_m \otimes A^{n \times n}) \right) &= (L_n^{mn} \otimes I_k) \left(I_m \otimes L_n^{kn} (I_k \otimes A^{n \times n}) \right) (L_m^{km} \otimes I_n) \\ I_k \otimes (A^{m \times m} \otimes I_n) &= (L_k^{km} \otimes I_n) \left((A^{m \times m} \otimes I_k) \otimes I_n \right) (L_m^{km} \otimes I_n) \end{aligned}$$