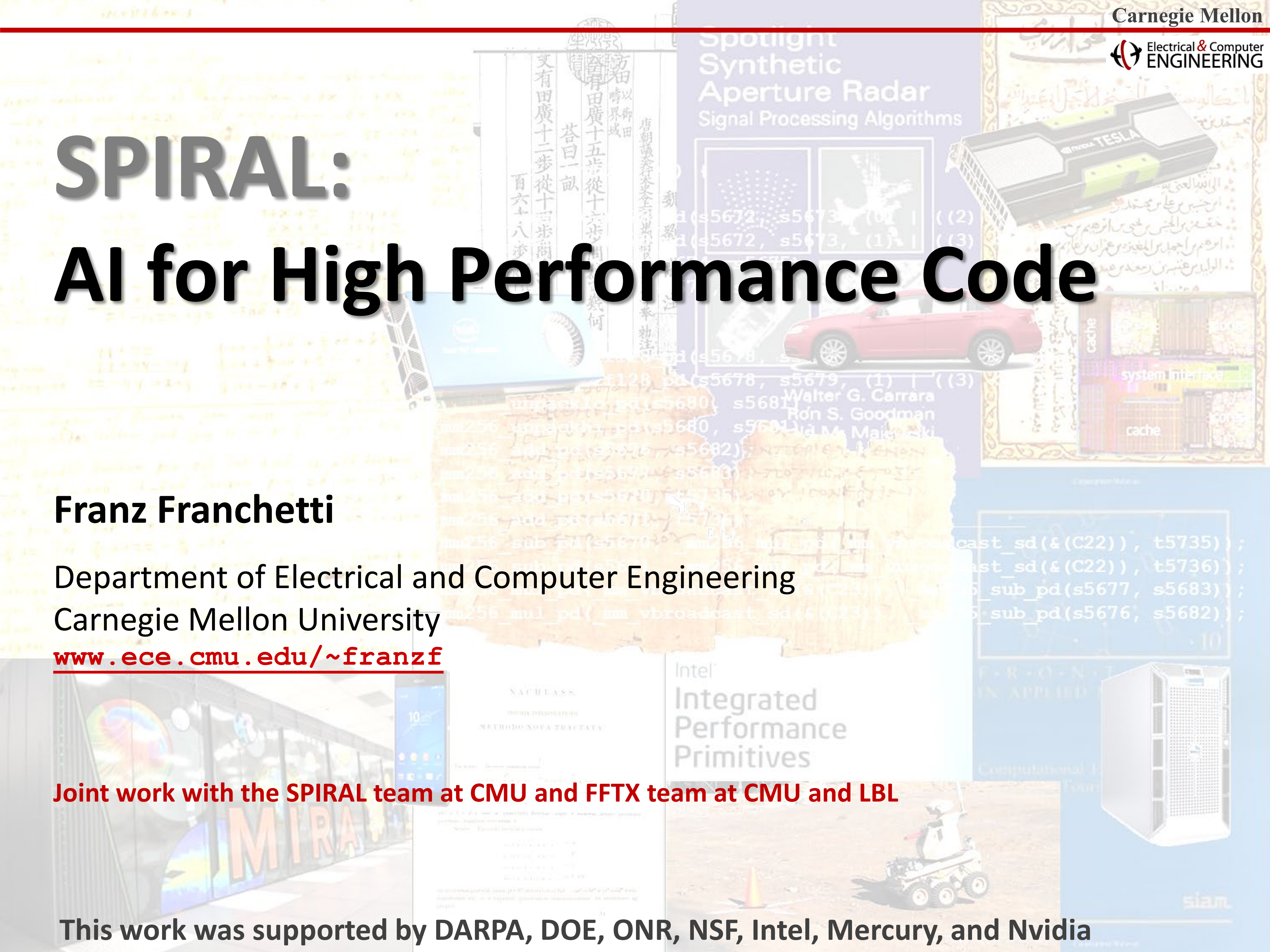
Spotlight Synthetic Aperture Radar
Signal Processing Algorithms
Walter G. Carrara
Ron S. Goodman
TESLA
system interface
cache
Intel
Integrated Performance Primitives
MIRA
siam

SPIRAL:

AI for High Performance Code

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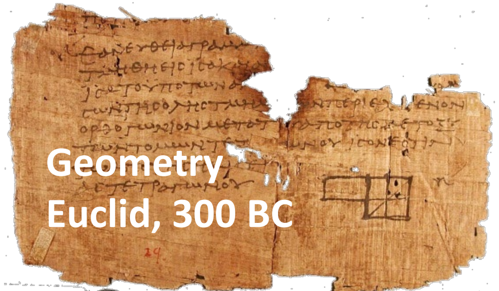
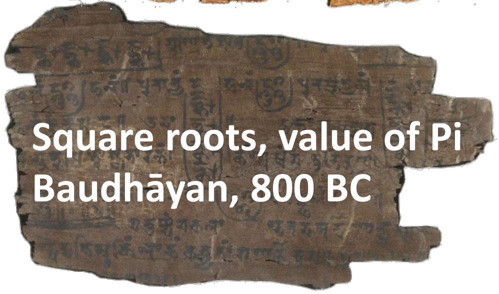
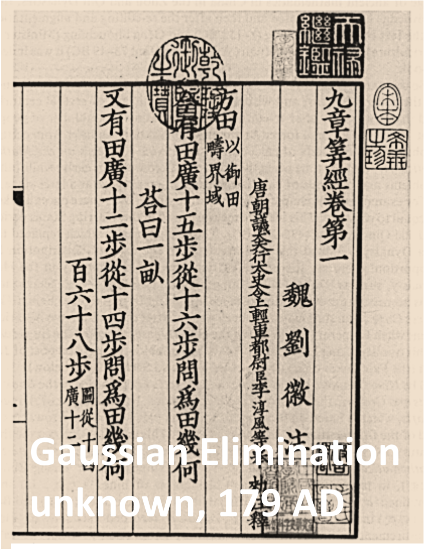
Joint work with the SPIRAL team at CMU and FFTX team at CMU and LBL

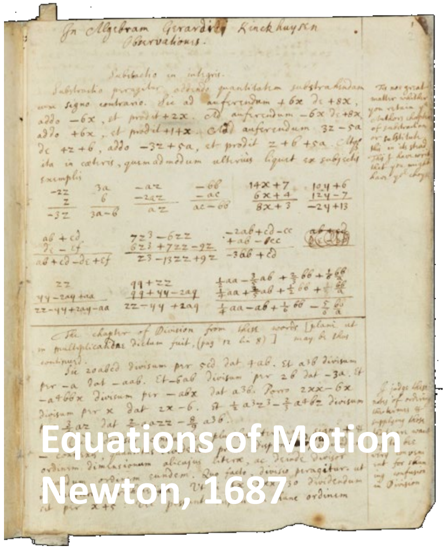
This work was supported by DARPA, DOE, ONR, NSF, Intel, Mercury, and Nvidia

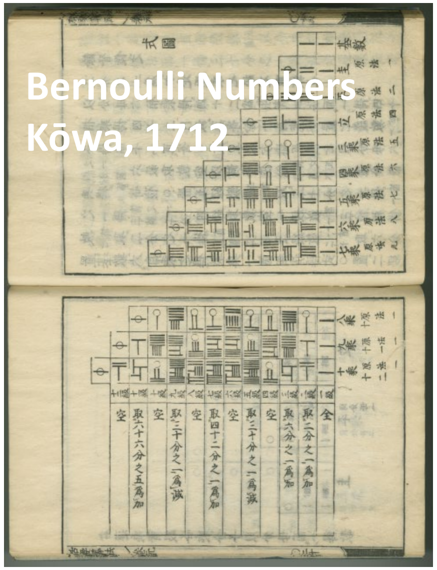
Algorithms and Mathematics: 2,500+ Years


Geometry
Euclid, 300 BC


Square roots, value of Pi
Baudhāyan, 800 BC

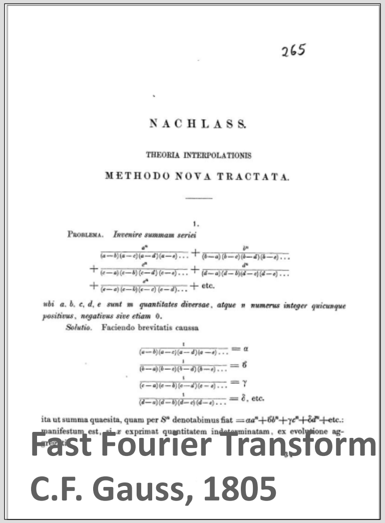

Gaussian Elimination
unknown, 179 AD


Equations of Motion
Newton, 1687

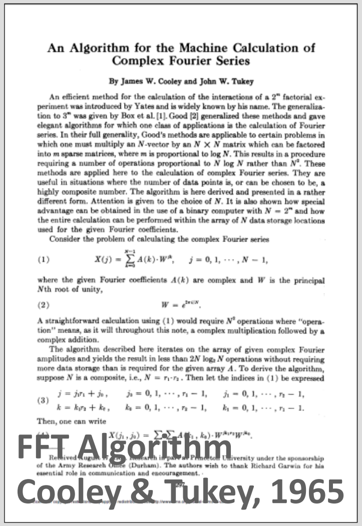

Bernoulli Numbers
Kōwa, 1712

Algebra
al-Khwārizmī, 830

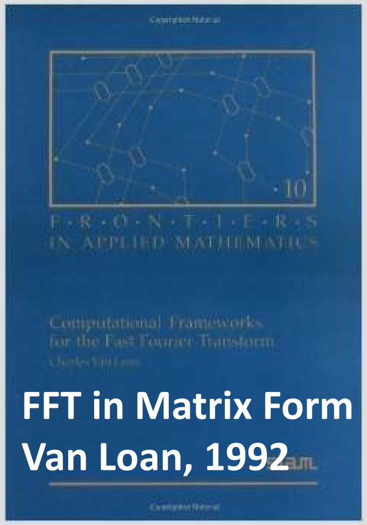
Fast Fourier Transform


NACHLASS.
THEORIA INTERPOLATIONIS
METHODO NOVA TRACTATA.

Fast Fourier Transform
C.F. Gauss, 1805


An Algorithm for the Machine Calculation of Complex Fourier Series
By James W. Cooley and John W. Tukey

FFT Algorithm
Cooley & Tukey, 1965


10
FRONTIERS IN APPLIED MATHEMATICS
Computational Frameworks for the Fast Fourier Transform
Charles Van Loan
FFT in Matrix Form
Van Loan, 1992

Computing Platforms Over The Years

F-16A/B, C/D, E/F, IN, IQ, N, V: Flying since 1974

Compare: Desktop/workstation class CPUs/machines

Assembly code compatible !!

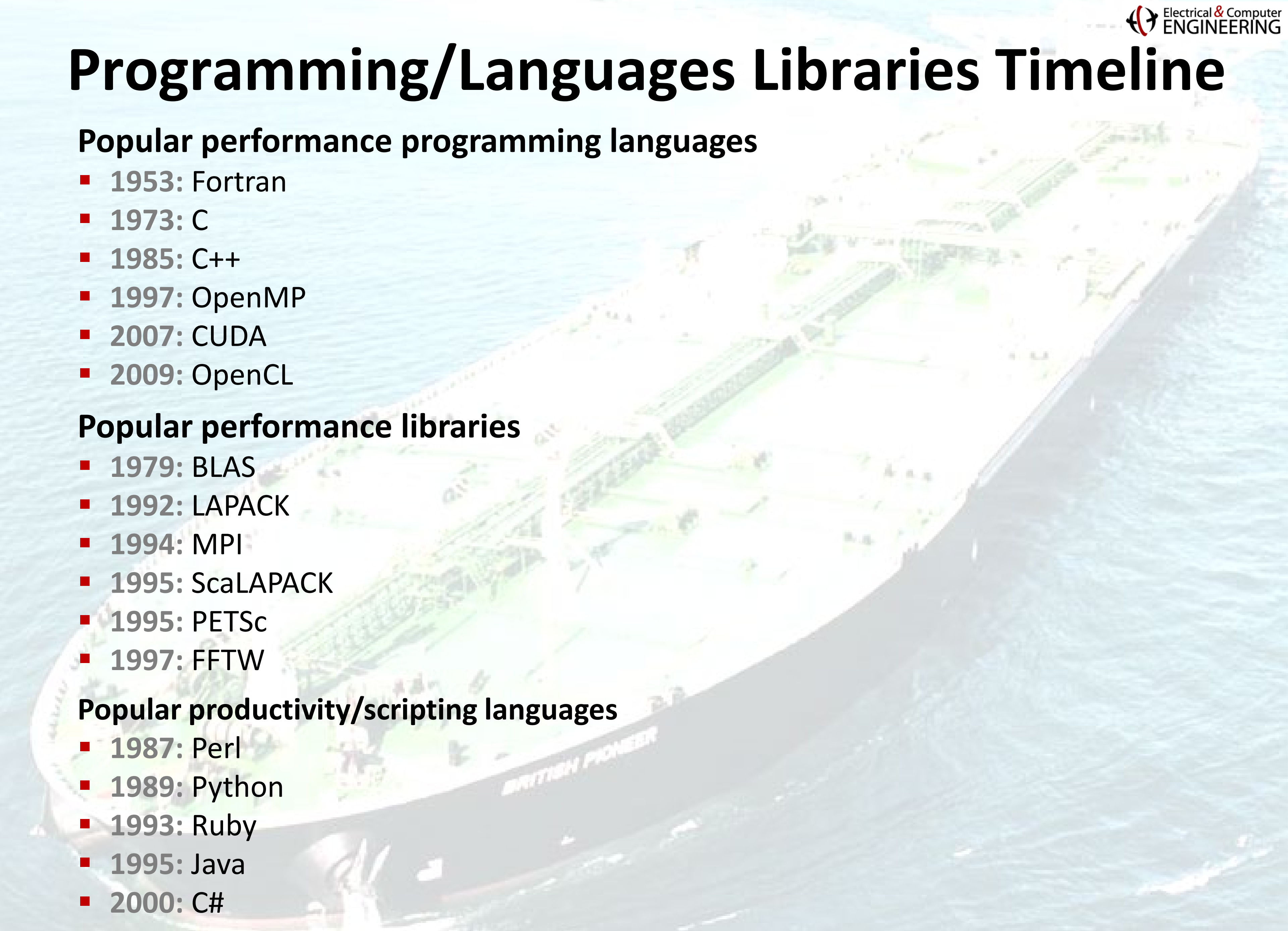
7

x86 binary compatible, but 500x parallelism ?!

1972	1989	1994	2006	2011	2018
Intel 8008	IBM PC/XT compatible	IBM RS/6000-390	GeForce 8800	Xeon Phi	Xeon Platinum 8180M
0.2—0.8 MHz	8088 @ 8 MHz, 640kB RAM	256 MB RAM, 6GB HDD	1.3 GHz, 128 shaders	1.3 GHz, 60 cores	28 cores, 2.5-3.6 GHz
Intelligent terminal	360 kB FDD, 720x348 mono	67 MHz Power2+, AIX	16-way SIMT	8/16-way SIMD	2/4/8/16-way SIMD

10^7 – 10^8 compounded performance gain over 45 years

Programming/Languages Libraries Timeline



Popular performance programming languages

- 1953: Fortran
- 1973: C
- 1985: C++
- 1997: OpenMP
- 2007: CUDA
- 2009: OpenCL

Popular performance libraries

- 1979: BLAS
- 1992: LAPACK
- 1994: MPI
- 1995: ScaLAPACK
- 1995: PETSc
- 1997: FFTW

Popular productivity/scripting languages

- 1987: Perl
- 1989: Python
- 1993: Ruby
- 1995: Java
- 2000: C#

2019: What $1M Can Buy You

Dell PowerEdge R940
4.5 Tflop/s, 6 TB, 850 W
4x 28 cores, 2.5 GHz

24U rack
7.5kW
<$1M

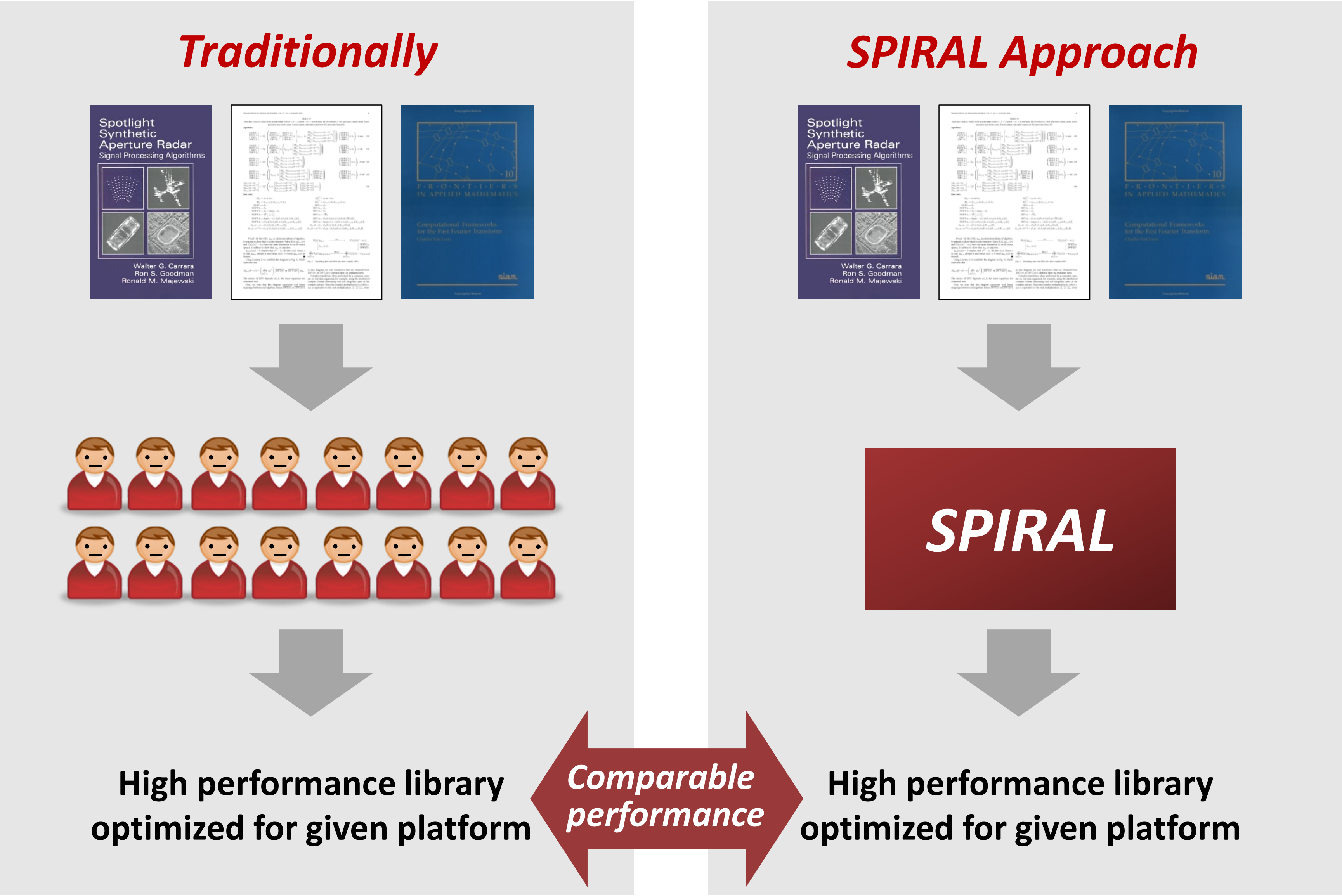
OSS FSAn-4
200 TB PCIe NVMe flash
80 GB/s throughput

BittWare TeraBox
18M logic elements, 4.9 Tb/sec I/O
8 FPGA cards/16 FPGAs, 2 TB DDR4

AberSAN ZXP4
90x 12TB HDD, 1 kW
1PB raw

Nvidia DGX-1
8x Tesla V100, 3.2 kW
170 Tflop/s, 128 GB

SPIRAL: AI for High Performance Code



Traditionally

High performance library optimized for given platform

SPIRAL Approach

SPIRAL

High performance library optimized for given platform

Comparable performance

Spotlight Synthetic Aperture Radar
Signal Processing Algorithms
Walter G. Carrara
Ron S. Goodman
Ronald M. Majewski
10
FRONTIERS IN APPLIED MATHEMATICS
Computational Frameworks for the Fast Fourier Transform
siam

Outline

- Introduction
- **Specifying computation**
- Achieving Performance Portability
- FFTX: A Library Frontend for SPIRAL
- Summary

F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura:
SPIRAL: Extreme Performance Portability, Proceedings of the IEEE, Vol. 106, No. 11, 2018.
Special Issue on *From High Level Specification to High Performance Code*

SPIRAL: AI for Performance Engineering

Given:

- **Mathematical problem specification**

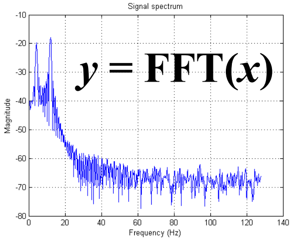
  core mathematics does not change

- **Target computer platform**

  varies greatly, new platforms introduced often

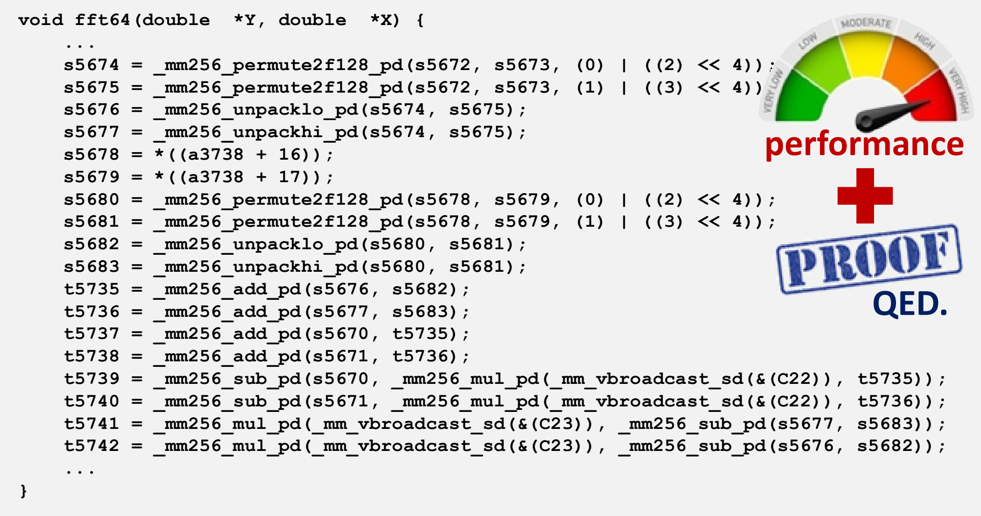
Wanted:

- **Very good implementation of specification on platform**
- **Proof of correctness**



on

automatic



```
void fft64(double  *Y, double  *X) {
    ...
    s5674 = _mm256_permute2f128_pd(s5672, s5673, (0) | ((2) << 4));
    s5675 = _mm256_permute2f128_pd(s5672, s5673, (1) | ((3) << 4))
    s5676 = _mm256_unpacklo_pd(s5674, s5675);
    s5677 = _mm256_unpackhi_pd(s5674, s5675);
    s5678 = *((a3738 + 16));
    s5679 = *((a3738 + 17));
    s5680 = _mm256_permute2f128_pd(s5678, s5679, (0) | ((2) << 4));
    s5681 = _mm256_permute2f128_pd(s5678, s5679, (1) | ((3) << 4));
    s5682 = _mm256_unpacklo_pd(s5680, s5681);
    s5683 = _mm256_unpackhi_pd(s5680, s5681);
    t5735 = _mm256_add_pd(s5676, s5682);
    t5736 = _mm256_add_pd(s5677, s5683);
    t5737 = _mm256_add_pd(s5670, t5735);
    t5738 = _mm256_add_pd(s5671, t5736);
    t5739 = _mm256_sub_pd(s5670, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5735));
    t5740 = _mm256_sub_pd(s5671, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5736));
    t5741 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5677, s5683));
    t5742 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5676, s5682));
    ...
}
```

OL Operators

Definition

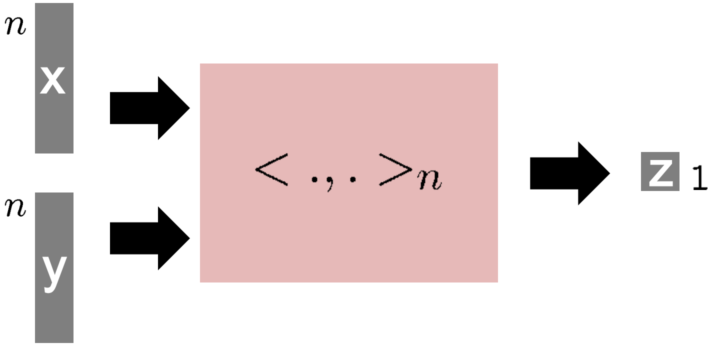
- Operator: Multiple vectors ! Multiple vectors
- Stateless
- Higher-dimensional data is linearized
- Operators are potentially nonlinear

$$\mathrm{M} : \begin{cases} \mathbb{C}^{n_0} \times \cdots \times \mathbb{C}^{n_{k-1}} \to \mathbb{C}^{N_0} \times \cdots \times \mathbb{C}^{N_{\ell-1}} \\ (\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}) \mapsto \mathrm{M}(\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}) \end{cases}$$

Example: Scalar product

$$< .,. >_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$

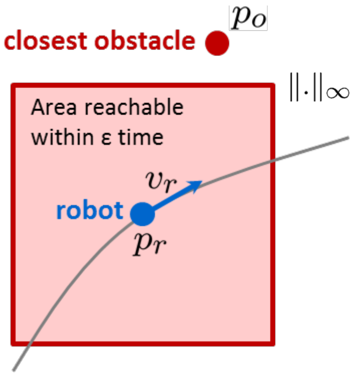
$$\big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto \sum_{i=0}^{n-1} x_i y_i$$



Example: Safety Distance as OL Operator

- **Passive Safety of Robots**

  p_o: Position of closest obstacle
  p_r: Position of robot
  v_r: Longitudinal velocity of robot
  A, b, V, ": constants



$$\|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r+V)\right)$$

- **Definition as operator**

$$\begin{aligned} \mathsf{SafeDist}_{V,A,b,\varepsilon} : \quad & \mathbb{R}\times\mathbb{R}^2\times\mathbb{R}^2 \to \mathbb{Z}_2 \\ & (v_r, p_r, p_o) \mapsto \big(p(v_r) < d_\infty(p_r, p_o)\big) \quad \text{with} \quad d_\infty(\vec{x},\vec{y}) = \|\vec{x}-\vec{y}\|_\infty \end{aligned}$$

$$p(x) = \alpha x^2 + \beta x + \gamma$$

$$\alpha = \tfrac{1}{2b}$$

$$\beta = \tfrac{V}{b} + \varepsilon\left(\tfrac{A}{b}+1\right)$$

$$\gamma = \left(\tfrac{A}{b}+1\right)\left(\tfrac{A}{2}\varepsilon^2 + \varepsilon V\right)$$

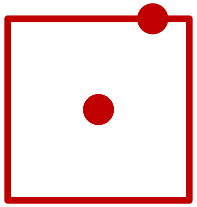
Formalizing Mathematical Objects in OL

- Infinity norm

$$\begin{aligned}\|.\|_\infty^n :\ & \mathbb{R}^n \to \mathbb{R}\\ & (x_i)_{i=0,\ldots,n-1} \mapsto \max_{i=0,\ldots,n-1} |x_i|\end{aligned}$$

- Chebyshev distance

$$\begin{aligned}d_\infty^n(.,.) :\ & \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}\\ & (x, y) \mapsto \|x - y\|_\infty^n\end{aligned}$$



- Vector subtraction

$$\begin{aligned}(-)_n :\ & \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n\\ & (x, y) \mapsto x - y\end{aligned}$$

- Pointwise comparison

$$\begin{aligned}(<)_n :\ & \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{Z}_2^n\\ & \big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto (x_i < y_i)_{i=0,\ldots,n-1}\end{aligned}$$

- Scalar product

$$\begin{aligned}< .,. >_n :\ & \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}\\ & \big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto \sum_{i=0}^{n-1} x_i y_i\end{aligned}$$

- Monomial enumerator

$$\begin{aligned}(x^i)_n :\ & \mathbb{R} \to \mathbb{R}^{n+1}\\ & x \mapsto (x^i)_{i=0,\ldots,n}\end{aligned}$$

- Polynomial evaluation

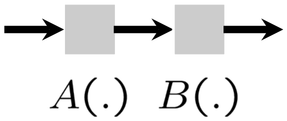
$$\begin{aligned}P[x, (a_0, \ldots, a_n)] :\ & \mathbb{R} \to \mathbb{R}\\ & x \mapsto a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n\end{aligned}$$

Beyond the textbook: explicit vector length, infix operators as prefix operators

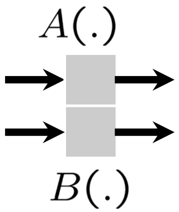
Operations and Operator Expressions

- **Operations (higher-order operators)**

$$\circ : (D \to S) \times (S \to R) \to (D \to R)$$
$$(A, B) \mapsto B \circ A$$

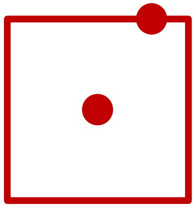


$$\times : (D \to R) \times (E \to S) \to (D \times E \to R \times S)$$
$$(A, B) \mapsto \Big((x, y) \mapsto \big(A(x), B(y)\big)\Big)$$



- **Operator expressions are operators**

$$\|.\|_\infty^n \circ (-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$\big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto \max_{i=0,\ldots,n-1} |x_i - y_i|$$



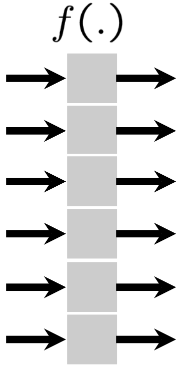
- **Short-hand notation: Infix notation**

$A(.) - B(.) = \big(x \mapsto A(x) - B(x)\big)$ can be expressed via

$$(-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$(x, y) \mapsto x - y$$

Basic OL Operators

- **Basic operators ≈ functional programming constructs**



map

$$\mathsf{Pointwise}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}^n$$
$$(x_i)_i \mapsto f_0(x_0) \oplus \cdots \oplus f_{n-1}(x_{n-1})$$

binop

$$\mathsf{Atomic}_{f(.,.)} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$
$$(x, y) \mapsto f(x, y)$$

map + zip

$$\mathsf{Pointwise}_{n\times n,f_i} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$\big((x_i)_i, (y_i)_i\big) \mapsto f_0(x_0, y_0) \oplus \cdots \oplus f_{n-1}(x_{n-1}, y_{n-1})$$

fold

$$\mathsf{Reduction}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}$$
$$(x_i)_i \mapsto f_{n-1}(x_{n-1}, f_{n-2}(x_{n-2}, f_{n-3}(\ldots f_0(x_0, \mathsf{id}())\ldots)$$

unfold

$$\mathsf{Induction}_{n,f_i} : \mathbb{R} \to \mathbb{R}^{n+1}$$
$$x \mapsto (f_n(x, f_{n-1}(\ldots)\ldots), \ldots, f_2(x, f_1(x, \mathsf{id})), f_1(x, \mathsf{id}), \mathsf{id}())$$

- **Safety distance as (optimized) operator expression**

$$\begin{aligned}\mathsf{SafeDist}_{V,A,b,\varepsilon} = \;& \mathsf{Atomic}_{(x,y)\mapsto x<y} \\ & \circ \Big(\Big(\mathsf{Reduction}_{3,(x,y)\mapsto x+y} \circ \mathsf{Pointwise}_{3,x\mapsto a_i x} \circ \mathsf{Induction}_{3,(a,b)\mapsto ab,1}\Big) \\ & \quad \times \Big(\mathsf{Reduction}_{2,(x,y)\mapsto \mathsf{max}(|x|,|y|)} \circ \mathsf{Pointwise}_{2\times 2,(x,y)\mapsto x-y}\Big)\Big)\end{aligned}$$

Breaking Down Operators into Expressions

- **Application specific:** Safety Distance as Rewrite Rule

$$\mathsf{SafeDist}_{V,A,b,\varepsilon}(.,.,.) \to \Big(P[x,(a_0,a_1,a_2)](.) < d_\infty^2(.,.)\Big)(.,.,.)$$

$$\text{with} \quad a_0{=}\tfrac{1}{2b},\ a_1{=}\tfrac{V}{b}{+}\varepsilon\Big(\tfrac{A}{b}{+}1\Big),\ a_2{=}\Big(\tfrac{A}{b}{+}1\Big)\Big(\tfrac{A}{2}\varepsilon^2{+}\varepsilon V\Big)$$

Problem specification: hand-developed or automatically produced

- **One-time effort:** mathematical library

$$d_\infty^n(.,.) \to \|.\|_\infty^n \circ (-)_n$$

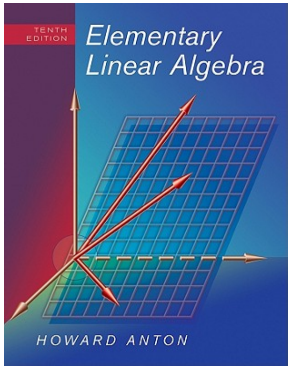
$$(\diamond)_n \to \mathsf{Pointwise}_{n\times n,(a,b)\mapsto a\diamond b}, \quad \diamond \in \{+,\ - \cdot,\ \wedge,\ \vee, \ldots\}$$

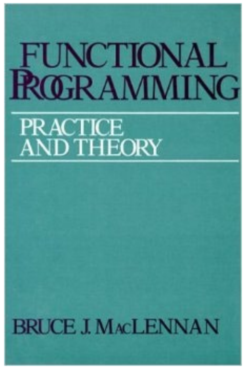
$$\|.\|_\infty^n \to \mathsf{Reduction}_{n,(a,b)\mapsto \mathsf{max}(|a|,|b|)}$$

$$< .,. >_n \to \mathsf{Reduction}_{n,(a,b)\mapsto a+b} \circ \mathsf{Pointwise}_{n\times n,(a,b)\mapsto ab}$$

$$P[x,(a_0,\ldots,a_n)] \to < (a_0,\ldots,a_n), . > \circ (x^i)_n$$

$$(x^i)_n \to \mathsf{Induction}_{n,(a,b)\mapsto ab,1}$$





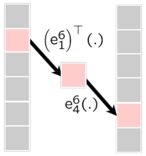
Library of well-known identities expressed in OL

Loop and Code Level Rule System

Mathematical Loop Abstraction

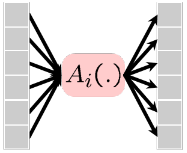
- **Selection and embedding operator:** *gather and scatter*

$$\left(\mathrm{e}_i^n\right)^\top(.) : \mathbb{R}^n \rightarrow \mathbb{R}^1$$
$$(x_i)_{i=0,\dots,n-1} \mapsto x_i$$
$$\mathrm{e}_i^n(.) : \mathbb{R}^1 \rightarrow \mathbb{R}^n$$
$$(x) \mapsto (0,\dots,0,\underbrace{x}_{i^{\text{th}}},0,\dots,0)$$



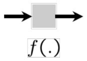
- **Iterative operations:** *loop*

$$\coprod_{i=0}^{n-1} : (D\rightarrow R)^n \rightarrow (D\rightarrow R)$$
$$A_i \mapsto (x \mapsto A_0(x) \sqcup \dots \sqcup A_{n-1}(x))$$
$$\text{with } \sqcup \in \left\{\sum, \vee, \wedge, \prod, \min, \max, \dots\right\}$$



- **Atomic operators:** *nonlinear scalar functions*

$$\mathrm{Atomic}_f : \mathbb{R}^1 \rightarrow \mathbb{R}^1$$
$$(x) \mapsto (f(x))$$

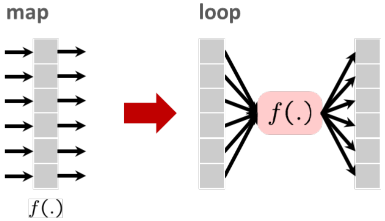


Translation and Optimization

- **Translating Basic OL into Σ-OL**

$$\mathrm{Pointwise}_{n,f_i} \rightarrow \sum_{i=0}^{n-1}\left(\mathrm{e}_i^n \circ \mathrm{Atomic}_{f_i} \circ \left(\mathrm{e}_i^n\right)^\top\right)$$

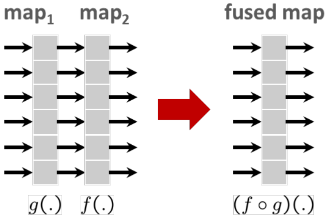
$$\mathrm{Reduction}_{n,(a,b)\mapsto a+b} \rightarrow \sum_{i=0}^{n-1}\left(\mathrm{e}_i^n\right)^\top$$



- **Optimizing Basic OL/Σ-OL**

$$\mathrm{Pointwise}_{n,f_i} \circ \mathrm{Pointwise}_{n,g_i} \rightarrow \mathrm{Pointwise}_{n,f_i\circ g_i}$$

$$\mathrm{Pointwise}_{n,f_i} \circ \mathrm{e}_n^j \rightarrow \mathrm{e}_n^j \circ \mathrm{Pointwise}_{1,f_j}$$



Abstract Code

Code objects

- Values and types
- Arithmetic operations
- Logic operations
- Constants, arrays and scalar variables
- Assignments and control flow

Properties: at the same time

- Program = (abstract syntax) tree
- Represents program in restricted C
- OL operator over real numbers and machine numbers (floating-point)
- Pure functional interpretation
- Represents lambda expression

```
# Dynamic Window Monitor

let(
  i3 := var("i3", TInt), i5 := var("i5", TInt),
  w2 := var("w2", TBool), w1 := var("w1", T_Real(64)),
  s8 := var("s8", T_Real(64)), s7 := var("s7", T_Real(64)),
  s6 := var("s6", T_Real(64)), s5 := var("s5", T_Real(64)),
  s4 := var("s4", T_Real(64)), s1 := var("s1", T_Real(64)),
  q4 := var("q4", T_Real(64)), q3 := var("q3", T_Real(64)),
  D := var("D", TPtr(T_Real(64)).aligned([16, 0])),
  X := var("X", TPtr(T_Real(64)).aligned([16, 0])),

  func(TInt, "dwmonitor", [ X, D ],
     decl([q3, q4, s1, s4, s5, s6, s7, s8, w1, w2],
        chain(
           assign(s5, V(0.0)),
           assign(s8, nth(X, V(0))),
           assign(s7, V(1.0)),
           loop(i5, [0..2],
              chain(
                 assign(s4, mul(s7, nth(D, i5))),
                 assign(s5, add(s5, s4)),
                 assign(s7, mul(s7, s8))
              )
           ),
           assign(s1, V(0.0)),
           loop(i3, [0..1],
              chain(
                 assign(q3, nth(X, add(i3, V(1)))),
                 assign(q4, nth(X, add(V(3), i3))),
                 assign(w1, sub(q3, q4)),
                 assign(s6, cond(geq(w1, V(0)), w1, neg(w1))),
                 assign(s1, cond(geq(s1, s6), s1, s6))
              )
           ),
           assign(w2, geq(s1, s5)),
           creturn(w2)
        )
     )
  )
)
```

Rule Based Compiler

Compilation rules: recursive descent

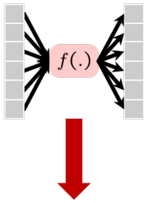
$$\mathrm{Code}\left(y = (A\circ B)(x)\right) \rightarrow \left\{\mathtt{decl}(t), \mathrm{Code}\left(t = B(x)\right), \mathrm{Code}\left(y = A(t)\right)\right\}$$

$$\mathrm{Code}\left(y = \left(\coprod_{i=0}^{n-1} A_i\right)(x)\right) \rightarrow \left\{y := \vec{0}, \mathtt{for}(i = 0..n-1)\ \mathrm{Code}\left(y \mathrel{+}= A_i(x)\right)\right\}$$

$$\mathrm{Code}\left(y = \left(\mathrm{e}_i^n\right)^\top(x)\right) \rightarrow y[0] := x[i]$$

$$\mathrm{Code}\left(y = \mathrm{e}_i^n(x)\right) \rightarrow \left\{y = \vec{0}, y[i] := x[0]\right\}$$

$$\mathrm{Code}\left(y = \mathrm{Atomic}_f(x)\right) \rightarrow y[0] := f(x[i])$$

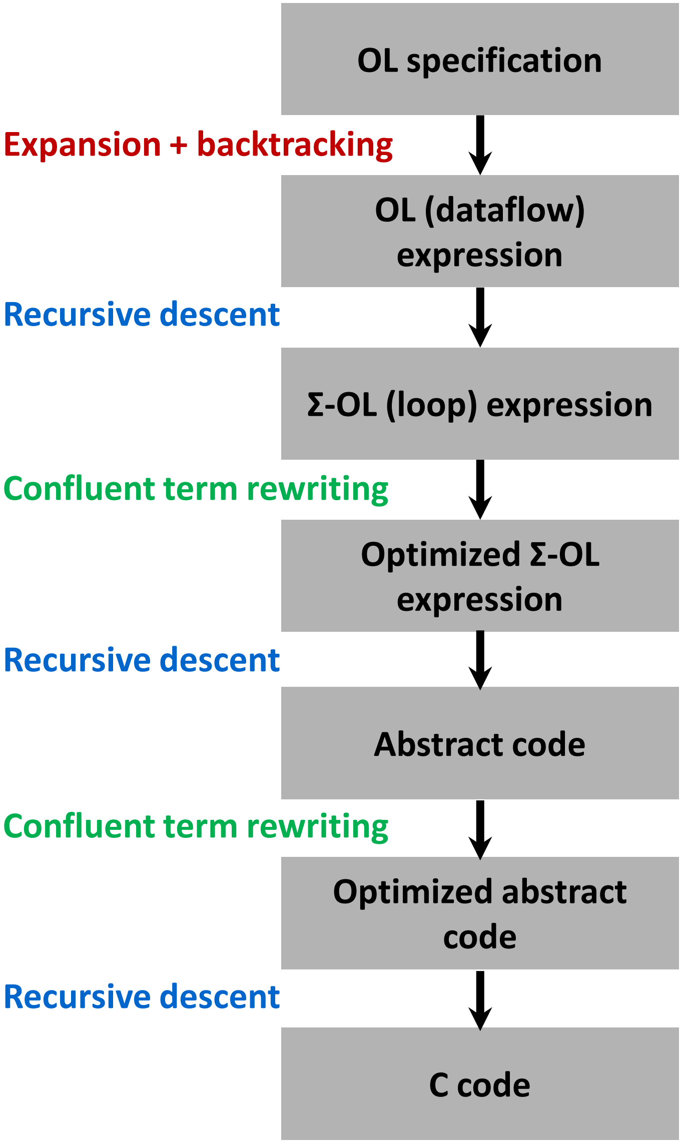


```
chain(
  assign(Y, V(0.0),
  loop(i1, [0..5],
    assign(nth(y, i1),
      f(nth(X, i1)))
  )
)
```

Cleanup rules: term rewriting

```
chain(a, chain(b))     → chain([a, b])
decl(D, decl(E, c))  → decl([D, E], c)
loop(i, decl(D, c))  → decl(D, loop(i, c))
chain(a, decl(D, b)) → decl(D, chain([a, b]))
```

Putting it Together: One Big Rule System

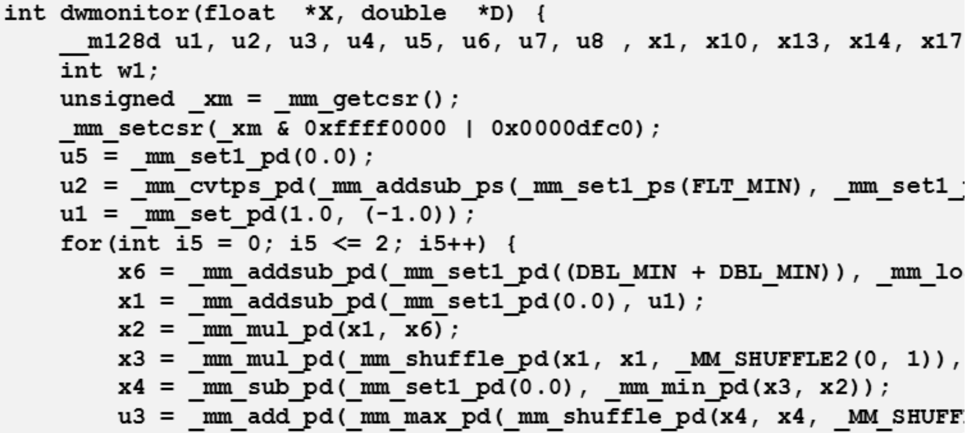


Mathematical specification

$$\mathsf{SafeDist}_{V,A,b,\varepsilon}(.,.,.) \rightarrow \Big(P[x,(a_0,a_1,a_2)](.) < d_\infty^2(.,.)\Big)(.,.,.)$$

with $a_0{=}\frac{1}{2b},\ a_1{=}\frac{V}{b}{+}\varepsilon\Big(\frac{A}{b}{+}1\Big),\ a_2{=}\Big(\frac{A}{b}{+}1\Big)\Big(\frac{A}{2}\varepsilon^2{+}\varepsilon V\Big)$

Final code



```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8 , x1, x10, x13, x14, x17
    int w1;
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN + DBL_MIN)), _mm_lo
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
        x3 = _mm_mul_pd(_mm_shuffle_pd(x1, x1, _MM_SHUFFLE2(0, 1)),
        x4 = _mm_sub_pd(_mm_set1_pd(0.0), _mm_min_pd(x3, x2));
        u3 = _mm_add_pd(_mm_max_pd(_mm_shuffle_pd(x4, x4, _MM_SHUFF
```

Final Synthesized C Code

```c
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8 , x1, x10, x13, x14, x17, x18, x19, x2, x3, x4, x6, x7, x8, x9;
    int w1;
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[0])));
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN + DBL_MIN)), _mm_loaddup_pd(&(D[i5])));
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
        x3 = _mm_mul_pd(_mm_shuffle_pd(x1, x1, _MM_SHUFFLE2(0, 1)), x6);
```

$$\mathsf{SafeDist}_{V,A,b,\varepsilon} = \mathsf{Atomic}_{(x,y)\mapsto x<y} \circ \Big(\Big(\mathsf{Reduction}_{3,(x,y)\mapsto x+y} \circ \mathsf{Pointwise}_{3,x\mapsto a_i x} \circ \mathsf{Induction}_{3,(a,b)\mapsto ab,1} \Big) \times \Big(\mathsf{Reduction}_{2,(x,y)\mapsto \max(|x|,|y|)} \circ \mathsf{Pointwise}_{2\times 2,(x,y)\mapsto x-y} \Big) \Big)$$

```c
    }
    u6
    for
        u8 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(i3 + 1)])));
        u7 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(3 + i3)])));
        x14 = _mm_add_pd(u8, _mm_shuffle_pd(u7, u7, _MM_SHUFFLE2(0, 1)));
        x13 = _mm_shuffle_pd(x14, x14, _MM_SHUFFLE2(0, 1));
        u4 = _mm_shuffle_pd(_mm_min_pd(x14, x13), _mm_max_pd(x14, x13), _MM_SHUFFLE2(1, 0));
        u6 = _mm_shuffle_pd(_mm_min_pd(u6, u4), _mm_max_pd(u6, u4), _MM_SHUFFLE2(1, 0));
    }
    x17 = _mm_addsub_pd(_mm_set1_pd(0.0), u6);
    x18 = _mm_addsub_pd(_mm_set1_pd(0.0), u5);
    x19 = _mm_cmpge_pd(x17, _mm_shuffle_pd(x18, x18, _MM_SHUFFLE2(0, 1)));
    w1 = (_mm_testc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffffff)) -
         (_mm_testnzc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffffff))));
    __asm nop;
    if (_mm_getcsr() & 0x0d) {
        _mm_setcsr(_xm);
        return -1;
    }
    _mm_setcsr(_xm);
    return w1;
}
```

Inspiration: Symbolic Integration

- **Rule based AI system**
  basic functions, substitution
- **May not succeed**
  not all expressions can be symbolically integrated
- **Arbitrarily extensible**
  define new functions as integrals
  Γ(.), distributions, Lebesgue integral
- **Semantics preserving**
  rule chain = formal proof
- **Automation**
  Mathematica, Maple

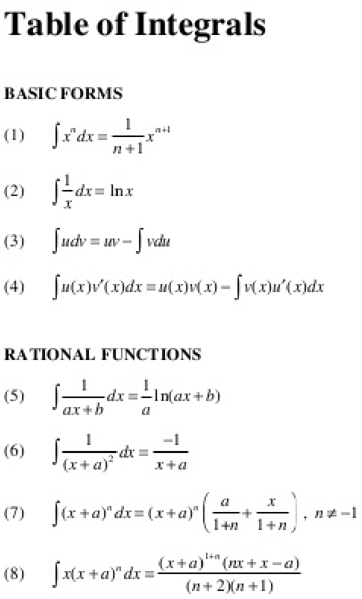


Table of Integrals

BASIC FORMS

(1) $\int x^n dx = \frac{1}{n+1} x^{n+1}$

(2) $\int \frac{1}{x} dx = \ln x$

(3) $\int u dv = uv - \int v du$

(4) $\int u(x) v'(x) dx = u(x) v(x) - \int v(x) u'(x) dx$

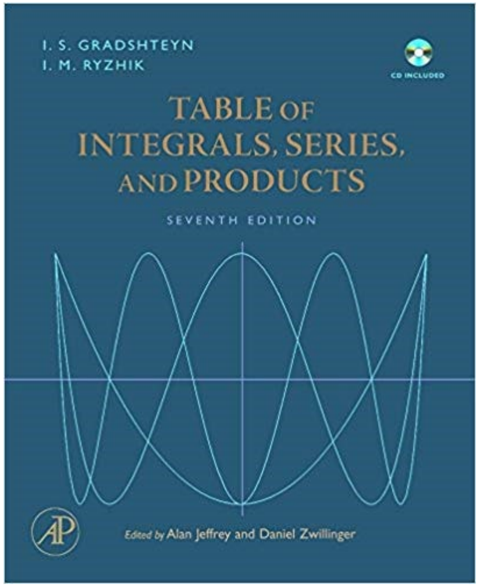
RATIONAL FUNCTIONS

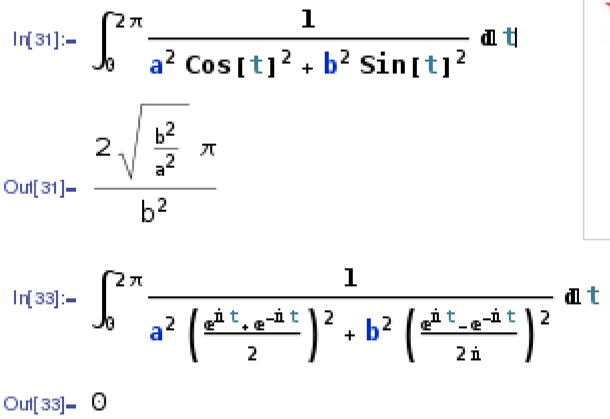
(5) $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax+b)$

(6) $\int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a}$

(7) $\int (x+a)^n dx = (x+a)^n \left(\frac{a}{1+n} + \frac{x}{1+n} \right), \ n \neq -1$

(8) $\int x(x+a)^n dx = \frac{(x+a)^{1+n}(nx+x-a)}{(n+2)(n+1)}$


I. S. GRADSHTEYN
I. M. RYZHIK
CD INCLUDED
TABLE OF INTEGRALS, SERIES, AND PRODUCTS
SEVENTH EDITION
AP
Edited by Alan Jeffrey and Daniel Zwillinger



In[31]:= $\int_0^{2\pi} \frac{1}{a^2 \, \mathrm{Cos}[t]^2 + b^2 \, \mathrm{Sin}[t]^2} \, dt$

Out[31]= $\frac{2 \sqrt{\frac{b^2}{a^2}} \, \pi}{b^2}$

In[33]:= $\int_0^{2\pi} \frac{1}{a^2 \left(\frac{e^{it} + e^{-it}}{2} \right)^2 + b^2 \left(\frac{e^{it} - e^{-it}}{2 i} \right)^2} \, dt$

Out[33]= 0

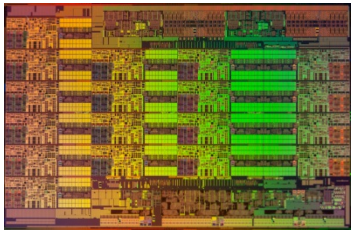
Outline

- Introduction
- Specifying computation
- **Achieving Performance Portability**
- FFTX: A Library Frontend for SPIRAL
- Summary

F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura:
SPIRAL: Extreme Performance Portability, Proceedings of the IEEE, Vol. 106, No. 11, 2018.
Special Issue on *From High Level Specification to High Performance Code*

Today’s Computing Landscape

1 Gflop/s = one billion floating-point operations (additions or multiplications) per second

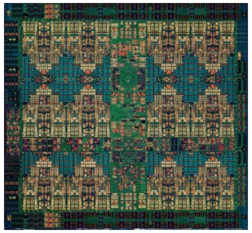


Intel Xeon 8180M

2.25 Tflop/s, 205 W

28 cores, 2.5—3.8 GHz

2-way—16-way AVX-512



IBM POWER9

768 Gflop/s, 300 W

24 cores, 4 GHz

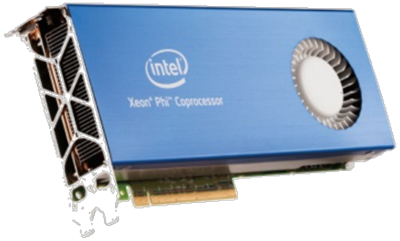
4-way VSX-3

Nvidia Tesla V100

7.8 Tflop/s, 300 W

5120 cores, 1.2 GHz

32-way SIMT



Intel Xeon Phi 7290F

1.7 Tflop/s, 260 W

72 cores, 1.5 GHz

8-way/16-way LRBni

Snapdragon 835

15 Gflop/s, 2 W

8 cores, 2.3 GHz

A540 GPU, 682 DSP, NEON

Intel Atom C3858

32 Gflop/s, 25 W

16 cores, 2.0 GHz

2-way/4-way SSSE3

Dell PowerEdge R940

3.2 Tflop/s, 6 TB, 850 W

4x 24 cores, 2.1 GHz

4-way/8-way AVX

Summit

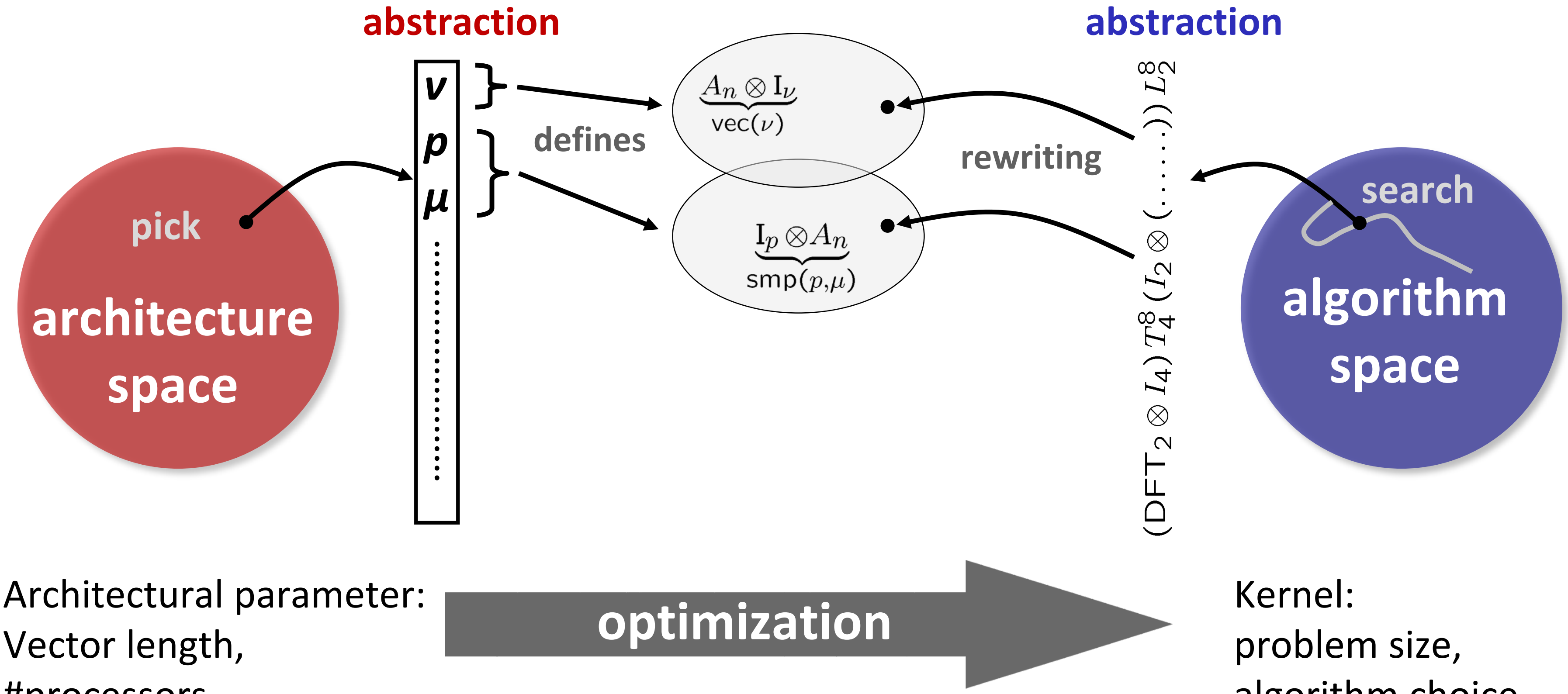
187.7 Pflop/s, 13 MW

9,216 x 22 cores POWER9

+ 27,648 V100 GPUs

Platform-Aware Formal Program Synthesis

Model: common abstraction
= spaces of matching formulas


abstraction
abstraction
ν
p
μ
defines
$\underbrace{A_n \otimes I_\nu}_{\mathrm{vec}(\nu)}$
$\underbrace{I_p \otimes A_n}_{\mathrm{smp}(p,\mu)}$
rewriting
$(\mathrm{DFT}_2 \otimes I_4)\, T^8_4\, (I_2 \otimes (\ldots\ldots))\, L^8_2$
pick
architecture space
search
algorithm space
optimization

Architectural parameter:
Vector length,
#processors, …

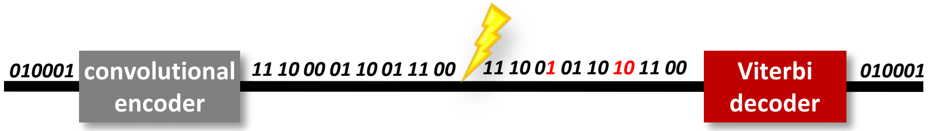
Kernel:
problem size,
algorithm choice

Some Application Domains in OL

Linear Transforms

$$
\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathsf{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathsf{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \mathsf{gcd}(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathsf{J}_m)\, \mathsf{L}^n_m (\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\qquad \cdot (\mathsf{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathsf{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathsf{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathsf{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathsf{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \right) \mathsf{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathsf{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathsf{diag}(1, 1/\sqrt{2})\, \mathsf{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathsf{J}_2\, \mathsf{R}_{13\pi/8}
\end{aligned}
$$

Software Defined Radio



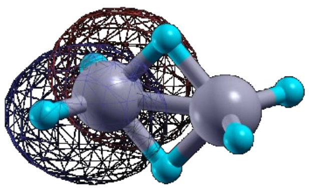
010001 convolutional encoder 11 10 00 01 10 01 11 00 → 11 10 01 01 10 10 11 00 Viterbi decoder 010001

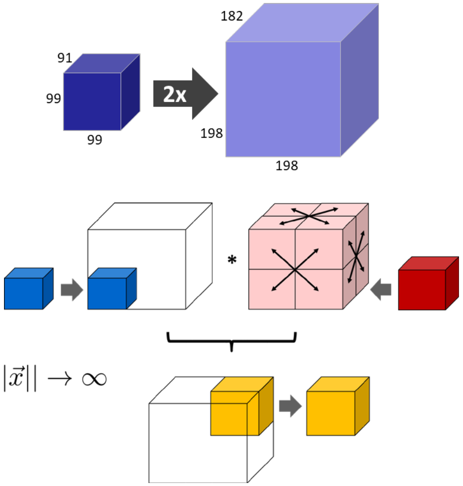
$$F_{K,F} \rightarrow \prod_{i=1}^{F} \left((I_{2^{K-2}} \otimes_j B_{F-i,j}) L^{2^{K-1}}_{2^{K-2}} \right)$$

$$\underline{\mathbf{F}_{K,F}}\ \nu \rightarrow \prod_{i=1}^{F} \left(\left(\mathrm{I}_{2^{K-2}/\nu} \otimes_{j_1} \vec{\mathsf{L}}^{2\nu}_{\nu} \vec{B}^{\nu}_{F-i,j_1} \right) (\mathsf{L}^{2^{K-1}/\nu}_{2^{K-2}/\nu} \bar{\otimes} \mathrm{I}_\nu) \right)$$

$$B_{i,j} : \begin{cases} \pi_U = \mathsf{min}_{d_U}(\pi_A + \beta_{A\rightarrow U}, \pi_B + \beta_{B\rightarrow U}) \\ \pi_V = \mathsf{min}_{d_V}(\pi_A + \beta_{A\rightarrow V}, \pi_B + \beta_{B\rightarrow V}) \end{cases}$$

PDEs/HPC Simulations



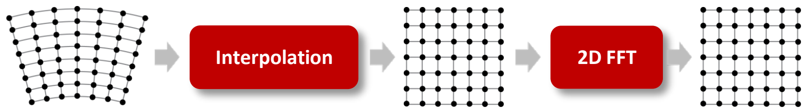


$$\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}$$

$$\Phi(\vec{x}) = \frac{Q}{4\pi||\vec{x}||} + o\left(\frac{1}{||\vec{x}||}\right) \text{ as } ||\vec{x}|| \rightarrow \infty$$

$$Q = \int_D \rho d\vec{x}$$

Synthetic Aperture Radar (SAR)



Interpolation → 2D FFT

$$
\begin{aligned}
\mathsf{SAR}_{k\times m\rightarrow n\times n} &\rightarrow \mathsf{DFT}_{n\times n} \circ \mathsf{Interp}_{k\times m\rightarrow n\times n} \\
\mathsf{DFT}_{n\times n} &\rightarrow (\mathsf{DFT}_n \otimes \mathrm{I}_n) \circ (\mathrm{I}_n \otimes \mathsf{DFT}_n) \\
\mathsf{Interp}_{k\times m\rightarrow n\times n} &\rightarrow (\mathsf{Interp}_{k\rightarrow n} \otimes_i \mathrm{I}_n) \circ (\mathrm{I}_k \otimes_i \mathsf{Interp}_{m\rightarrow n}) \\
\mathsf{Interp}_{r\rightarrow s} &\rightarrow \left(\bigoplus_{i=0}^{n-2} \mathsf{InterpSeg}_k \right) \oplus \mathsf{InterpSegPruned}_{k,\ell} \\
\mathsf{InterpSeg}_k &\rightarrow \mathsf{G}_f^{u\cdot n\rightarrow k} \circ \mathsf{iPrunedDFT}_{n\rightarrow u\cdot n} \circ \left(\frac{1}{n}\right) \circ \mathsf{DFT}_n
\end{aligned}
$$

Formal Approach for all Types of Parallelism

- **Multithreading** (Multicore)

$$\mathrm{I}_p \otimes_{\|} A_{\mu n}, \qquad \mathbf{L}_m^{mn} \bar{\otimes}\, \mathrm{I}_\mu$$

- **Vector SIMD** (SSE, VMX/Altivec,…)

$$A \hat{\otimes}\, \mathrm{I}_\nu \qquad \underbrace{\mathbf{L}_2^{2\nu}}_{\text{isa}}, \qquad \underbrace{\mathbf{L}_\nu^{2\nu}}_{\text{isa}}, \qquad \underbrace{\mathbf{L}_\nu^{\nu^2}}_{\text{isa}}$$

- **Message Passing** (Clusters, MPP)

$$\mathrm{I}_p \otimes_{\|} A_n, \qquad \underbrace{\mathbf{L}_p^{p^2} \bar{\otimes}\, \mathrm{I}_{n/p^2}}_{\text{all-to-all}}$$

- **Streaming/multibuffering** (Cell)

$$\mathrm{I}_n \otimes_2 A_{\mu n}, \qquad \mathbf{L}_m^{mn} \bar{\otimes}\, \mathrm{I}_\mu$$

- **Graphics Processors** (GPUs)

$$\prod_{i=0}^{n-1} A_i, \qquad A_n \hat{\otimes}\, \mathrm{I}_w, \qquad P_n \otimes Q_w$$

- **Gate-level parallelism** (FPGA)

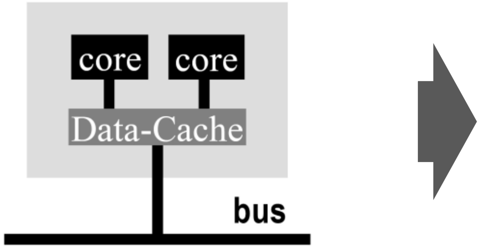
$$\prod_{i=0}^{n-1}{}^{\text{ir}} A, \qquad \mathrm{I}_s \tilde{\otimes} A, \qquad \underbrace{\mathbf{L}_n^m}_{\text{bram}}$$

- **HW/SW partitioning** (CPU + FPGA)

$$\underbrace{A_1}_{\text{fpga}}, \qquad \underbrace{A_2}_{\text{fpga}}, \qquad \underbrace{A_3}_{\text{fpga}}, \qquad \underbrace{A_4}_{\text{fpga}}$$

Modeling Hardware: Base Cases

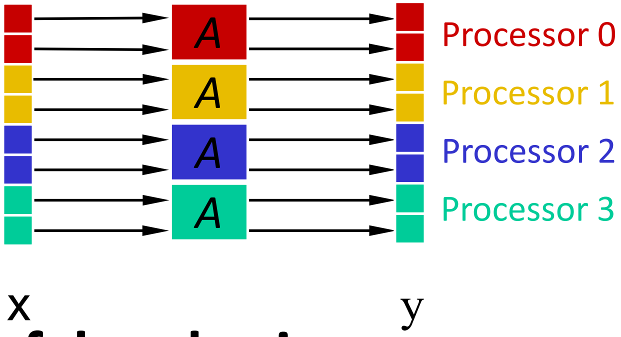
- Hardware abstraction: shared cache with cache lines



$$\underbrace{A_{k\times m\to n}}_{\mathrm{smp}(p,\mu)}$$

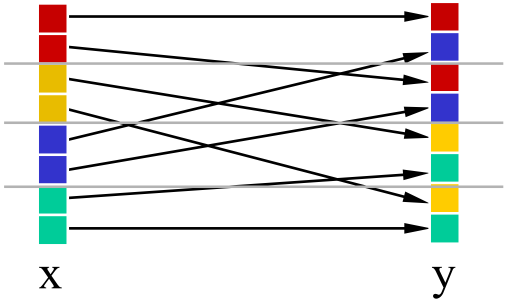
- Tensor product: embarrassingly parallel operator

$$y = \Big(\mathrm{I}_p \otimes A\Big)(x)$$



- Permutation: problematic; may produce false sharing

$$y = \mathrm{L}^8_4(x)$$



Example Program Transformation Rule Set

$$\underbrace{AB}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{A}_{\mathsf{smp}(p,\mu)} \underbrace{B}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{A_m \otimes \mathrm{I}_n}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{\left(\mathsf{L}^{mp}_m \otimes \mathrm{I}_{n/p}\right)\left(\mathrm{I}_p \otimes (A_m \otimes \mathrm{I}_{n/p})\right)\left(\mathsf{L}^{mp}_p \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{\mathsf{L}^{mn}_m}_{\mathsf{smp}(p,\mu)} \rightarrow \begin{cases} \underbrace{\left(\mathrm{I}_p \otimes \mathsf{L}^{mn/p}_{m/p}\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathsf{L}^{pn}_p \otimes \mathrm{I}_{m/p}\right)}_{\mathsf{smp}(p,\mu)} \\ \underbrace{\left(\mathsf{L}^{pm}_m \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathrm{I}_p \otimes \mathsf{L}^{mn/p}_m\right)}_{\mathsf{smp}(p,\mu)} \end{cases}$$

Recursive rules

$$\underbrace{\mathrm{I}_m \otimes A_n}_{\mathsf{smp}(p,\mu)} \rightarrow \mathrm{I}_p \otimes_{\|} \left(\mathrm{I}_{m/p} \otimes A_n\right)$$

$$\underbrace{(P \otimes \mathrm{I}_n)}_{\mathsf{smp}(p,\mu)} \rightarrow \left(P \otimes \mathrm{I}_{n/\mu}\right) \overline{\otimes}\, \mathrm{I}_\mu$$

Base case rules

Autotuning in Constraint Solution Space

AVX 2-way
`_Complex double`

$$\underbrace{\mathrm{DFT}_8}_{\mathrm{AVX}(2\text{-way }\mathbb{C})}$$

DFT$_8$

Base cases

$A^{n\times n}\vec{\otimes}\mathrm{I}_2$

$\underbrace{\mathbf{L}_2^4}_{\mathrm{vec}(2)}$

$\underbrace{\mathbf{T}_n^{mn}}_{\mathrm{vec}(2)}$

Transformation rules

$$(\mathrm{I}_m \otimes A^{n\times n})\mathbf{L}_m^{mn} \rightarrow \Big(\mathrm{I}_{m/\nu} \otimes \mathbf{L}_\nu^{n\nu}(A^{n\times n} \otimes \mathrm{I}_\nu)\Big)(\mathbf{L}_{m/\nu}^{mn/\nu} \otimes \mathrm{I}_\nu)$$

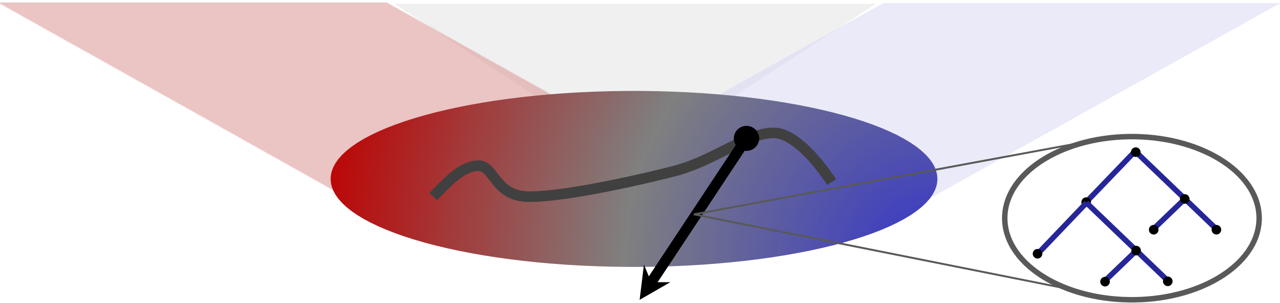
$$\mathbf{L}_\nu^{n\nu} \rightarrow (\mathbf{L}_\nu^n \otimes \mathrm{I}_\nu)(\mathrm{I}_{n/\nu} \otimes \mathbf{L}_\nu^{\nu^2})$$

$$A^{m\times m} \otimes \mathrm{I}_n \rightarrow (A^{m\times m} \otimes \mathrm{I}_{n/\nu}) \otimes \mathrm{I}_\nu$$

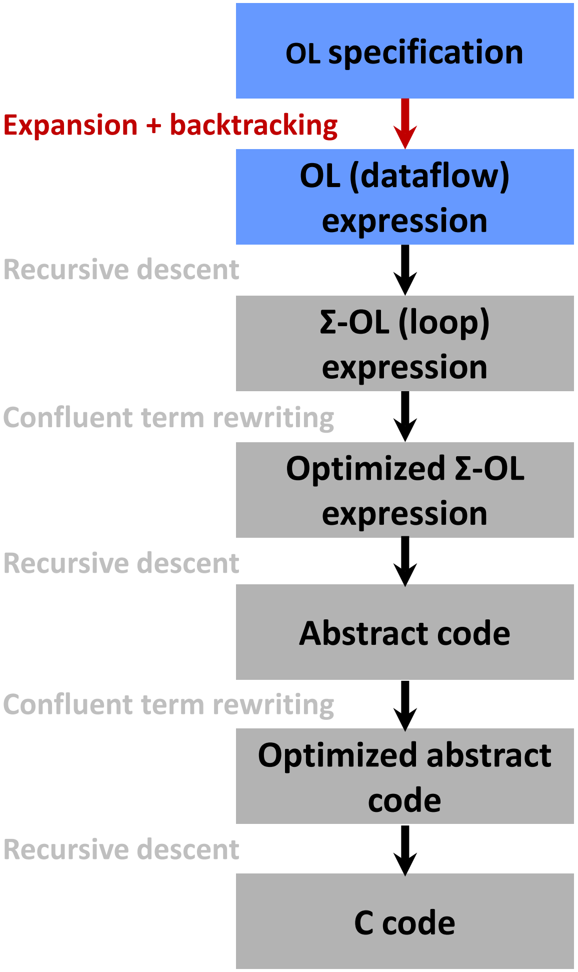
Breakdown rules

$$\mathbf{DFT}_{mn} \rightarrow (\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathbf{T}_n^{mn}(\mathrm{I}_m \otimes \mathbf{DFT}_n)\mathbf{L}_m^{mn}$$

$$\mathbf{DFT}_2 \rightarrow \mathrm{F}_2$$



$$\Big((\mathbf{F}_2 \otimes \mathrm{I}_2)\mathbf{T}_2^4(\mathrm{I}_2 \otimes \mathbf{F}_2)\mathbf{L}_2^4 \vec{\otimes}\, \mathrm{I}_2\Big)\ \underbrace{\mathbf{T}_2^8}_{\mathrm{vec}(2)}\ \Big(\mathrm{I}_2 \otimes\ \underbrace{\mathbf{L}_2^4}_{\mathrm{vec}(2)}\ (\mathbf{F}_2 \vec{\otimes}\, \mathrm{I}_2)\Big)\Big(\mathbf{L}_2^4 \vec{\otimes}\, \mathrm{I}_2\Big)$$



OL specification

Expansion + backtracking

OL (dataflow) expression

Recursive descent

Σ-OL (loop) expression

Confluent term rewriting

Optimized Σ-OL expression

Recursive descent

Abstract code

Confluent term rewriting

Optimized abstract code

Recursive descent

C code

Translating an OL Expression Into Code

Constraint Solver Input: $\underbrace{\mathsf{DFT}_8}_{\mathsf{AVX(2\text{-}way}\ \mathbb{C})}$

Output =

Ruletree, expanded into

OL Expression:

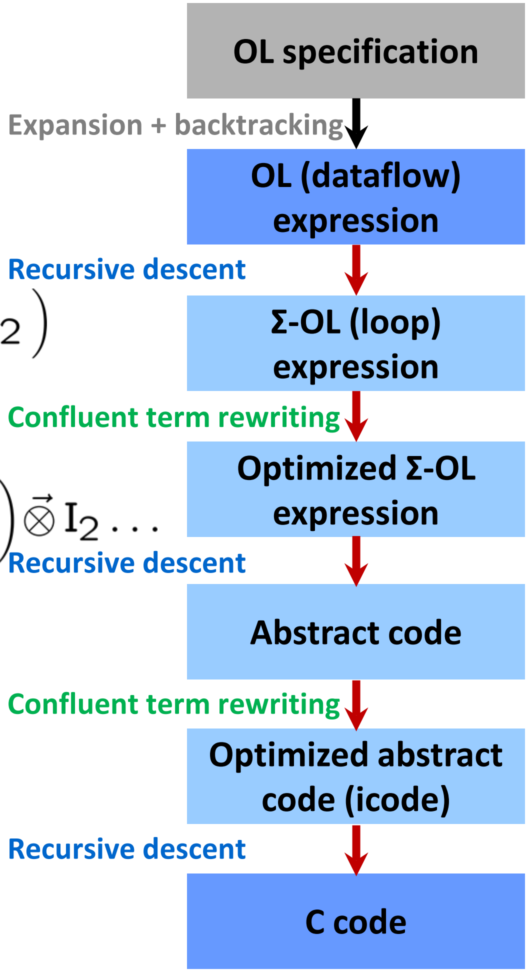
$$\Big(\big(\mathbf{F}_2\otimes\mathbf{I}_2\big)\mathbf{T}_2^4\big(\mathbf{I}_2\otimes\mathbf{F}_2\big)\mathbf{L}_2^4\vec{\otimes}\,\mathbf{I}_2\Big)\ \underbrace{\mathbf{T}_2^8}_{\mathsf{vec}(2)}\ \Big(\mathbf{I}_2\otimes\ \underbrace{\mathbf{L}_2^4}_{\mathsf{vec}(2)}\ \big(\mathbf{F}_2\vec{\otimes}\,\mathbf{I}_2\big)\Big)\Big(\mathbf{L}_2^4\vec{\otimes}\,\mathbf{I}_2\Big)$$

∑-OL:

$$\Bigg(\sum_{\jmath=0}^{1}\Big(\mathbf{S}_{\imath_2\otimes(\jmath)_2}\mathbf{F}_2\mathbf{Map}^2_{x\mapsto\omega_4^{2i+j}x}\mathbf{G}_{\imath_2\otimes(\jmath)_2}\Big)\sum_{\jmath=0}^{1}\Big(\mathbf{S}_{(\jmath)_2\otimes\imath_2}\mathbf{F}_2\mathbf{G}_{\imath_2\otimes(\jmath)_2}\Big)\Bigg)\vec{\otimes}\,\mathbf{I}_2\ldots$$

C Code:

```
void dft8(_Complex double *Y, _Complex double *X) {
    __m256d s38, s39, s40, s41,...
    __m256d  *a17, *a18;
    a17 = ((__m256d  *) X);
    s38 = *(a17);
    s39 = *((a17 + 2));
    t38 = _mm256_add_pd(s38, s39);
    t39 = _mm256_sub_pd(s38, s39);
    ...
    s52 = _mm256_sub_pd(s45, s50);
    *((a18 + 3)) = s52;
}
```



OL specification

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Abstract code

Confluent term rewriting

Optimized abstract code (icode)

Recursive descent

C code

Symbolic Verification for Linear Operators

- Linear operator = matrix-vector product
  Algorithm = matrix factorization

$$\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}=\begin{bmatrix}1&\cdot&1&\cdot\\\cdot&1&\cdot&1\\1&\cdot&-1&\cdot\\\cdot&1&\cdot&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&\cdot&\cdot&j\end{bmatrix}\begin{bmatrix}1&1&\cdot&\cdot\\1&-1&\cdot&\cdot\\\cdot&\cdot&1&1\\\cdot&\cdot&1&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&\cdot&1\end{bmatrix}\quad = ?$$

$$\mathrm{DFT}_4 = (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\mathrm{T}^4_2(\mathrm{I}_2 \otimes \mathrm{DFT}_2)\,\mathrm{L}^4_2$$

- Linear operator = matrix-vector product
  Program = matrix-vector product

$$\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\quad = ?$$

```
DFT4([0,1,0,0])
```

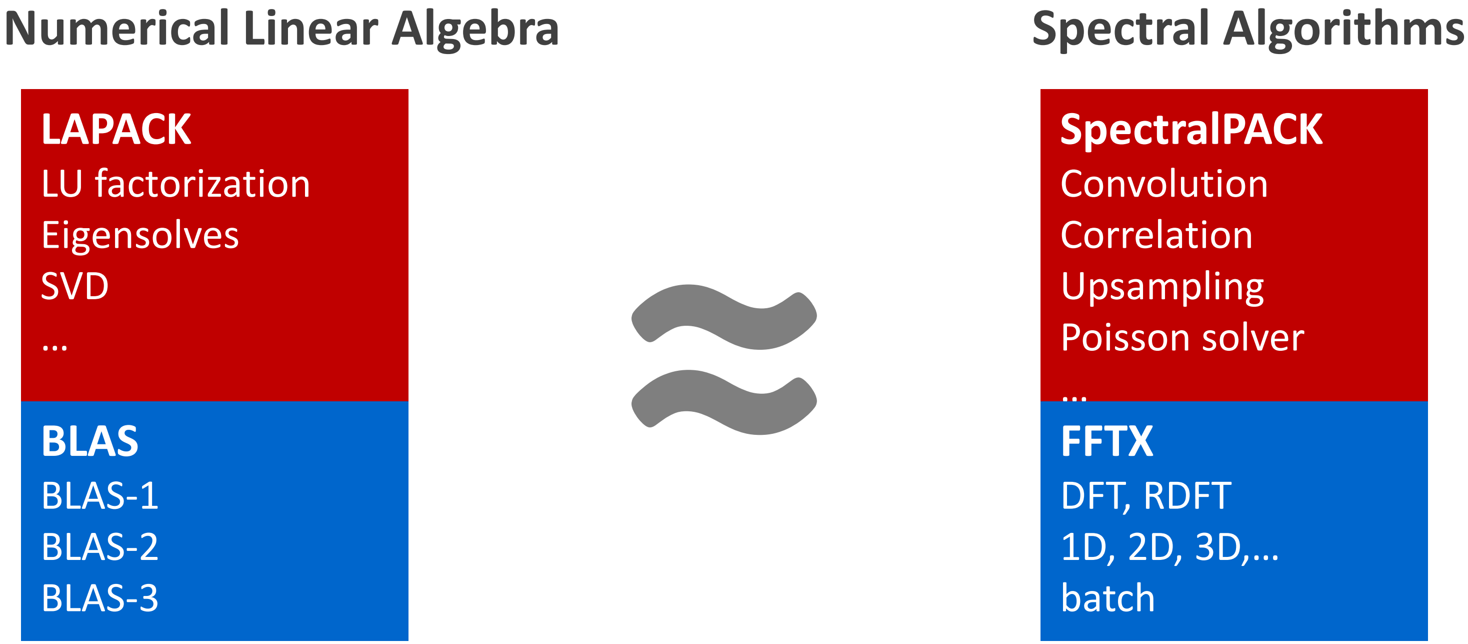
Symbolic evaluation and symbolic execution establishes correctness

Outline

- Introduction
- Specifying computation
- Achieving Performance Portability
- **FFTX: A Library Frontend for SPIRAL**
- Summary

F. Franchetti, D. G. Spampinato, A. Kulkarni, D. T. Popovici, T. M. Low, M. Franusich, A. Canning, P. McCorquodale, B. Van Straalen, P. Colella: **FFTX and SpectralPack: A First Look**, Workshop on Parallel Fast Fourier Transforms (PFFT).

FFTX and SpectralPACK



Numerical Linear Algebra

LAPACK
LU factorization
Eigensolves
SVD
…

BLAS
BLAS-1
BLAS-2
BLAS-3

≈

Spectral Algorithms

SpectralPACK
Convolution
Correlation
Upsampling
Poisson solver
…

FFTX
DFT, RDFT
1D, 2D, 3D,…
batch

Define the LAPACK equivalent for spectral algorithms

- **Define FFTX as the BLAS equivalent**
  provide user FFT functionality as well as algorithm building blocks
- **Define class of numerical algorithms to be supported by SpectralPACK**
  PDE solver classes (Green's function, sparse in normal/k space,…), signal processing,…
- **Library front-end, code generation and vendor library back-end**
  mirror concepts from FFTX layer

FFTX and SpectralPACK solve the "spectral motif" long term

Example: Poisson's Equation in Free Space

Partial differential equation (PDE)

$$\Delta(\Phi) = \rho$$

$$\rho : \mathbb{R}^3 \to \mathbb{R}$$

$$D = \mathrm{supp}(\rho) \subset \mathbb{R}^3$$

Poisson's equation. Δ is the Laplace operator

Solution characterization

$$\Phi : \mathbb{R}^3 \to \mathbb{R}$$

$$\Phi(\vec{x}) = \frac{Q}{4\pi||\vec{x}||} + o\left(\frac{1}{||\vec{x}||}\right) \quad \text{as } ||\vec{x}|| \to \infty$$

$$Q = \int_D \rho d\vec{x}$$

Approach: Green's function

$$\Phi(\vec{x}) = \int_D G(\vec{x} - \vec{y})\rho(\vec{y})d\vec{y} \equiv (G * \rho)(\vec{x}), \quad G(\vec{x}) = \frac{1}{4\pi||\vec{x}||_2}$$

Solution: ϕ(.) = convolution of RHS *ρ*(.) with Green's function *G*(.). Efficient through FFTs (frequency domain)

Method of Local Corrections (MLC)

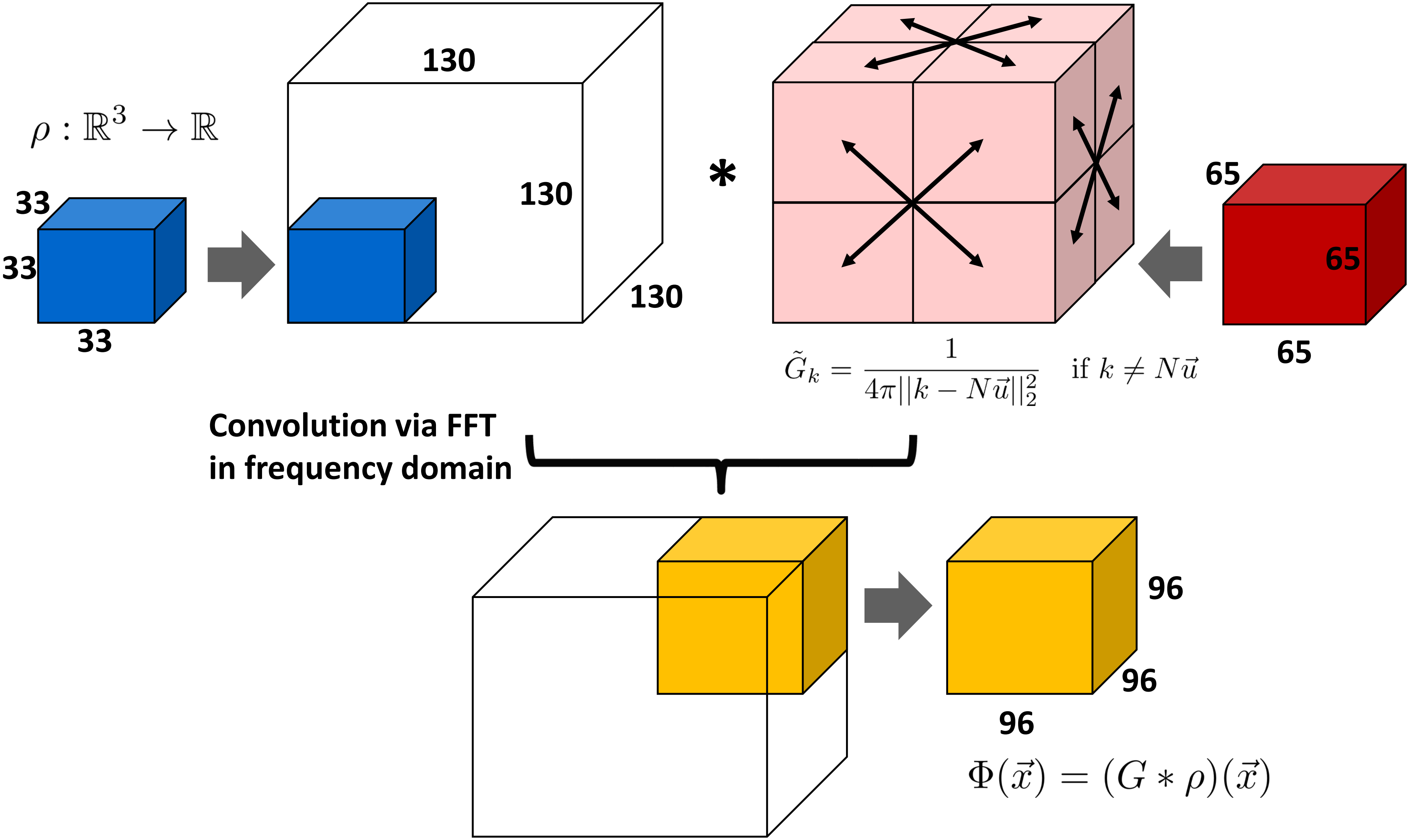
$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

Green's function kernel in frequency domain

P. McCorquodale, P. Colella, G. T. Balls, and S. B. Baden: **A Local Corrections Algorithm for Solving Poisson's Equation in Three Dimensions.** Communications in Applied Mathematics and Computational Science Vol. 2, No. 1 (2007), pp. 57-81., 2007.

C. R. Anderson: **A method of local corrections for computing the velocity field due to a distribution of vortex blobs.** Journal of Computational Physics, vol. 62, no. 1, pp. 111–123, 1986.

Algorithm: Hockney Free Space Convolution



$\rho : \mathbb{R}^3 \to \mathbb{R}$

$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

Convolution via FFT in frequency domain

$$\Phi(\vec{x}) = (G * \rho)(\vec{x})$$

Hockney: Convolution + problem specific zero padding and output subset

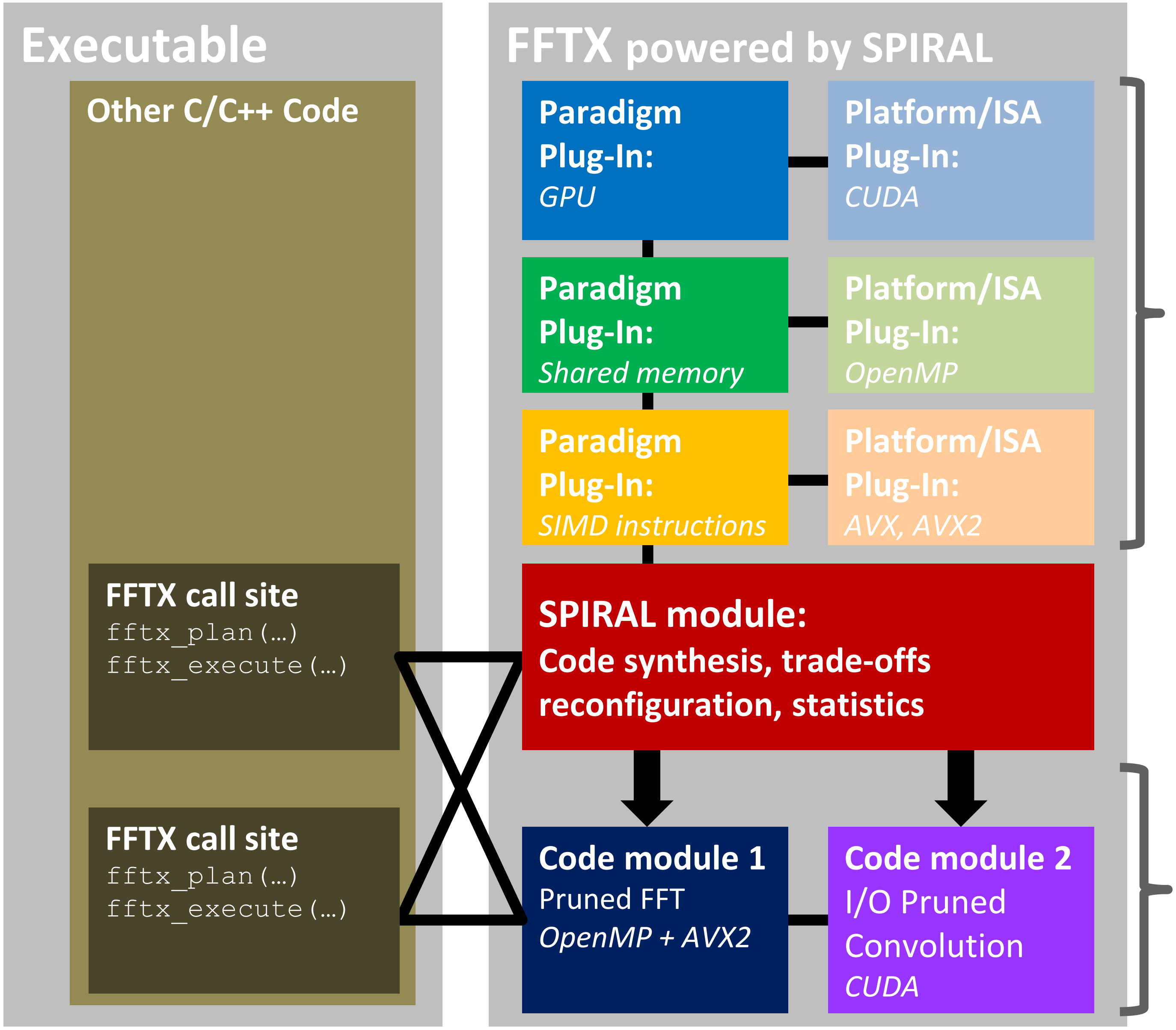
FFTX C++ Code: Hockney Free Space Convolution

```
box_t<3> inputBox(point_t<3>({{0,0,0}}),point_t<3>({32,32,32}));
array_t<3, double> rho(inputBox);
// ... set input values.

box_t<3> transformBox(point_t<3>({{0,0,0}}),point_t<3>({{129,129,129}}));
box_t<3> outputBox(point_t<3>({33,33,33}),point_t<3>({129,129,129}));

point_t<3> kindF({{DFT,DFT,DFT}});

size_t

auto fo
    pla

auto ke                                                                     lize);

point_t
auto in


auto so


context
context

std::ofstream splFile("hockney.spl");
export_spl(context, solver, splFile, "hockney33_97_130");
splFile.close();
// Offline codegen.
auto fptr = import_spl<3, double, double>("hockney33_97_130");
array_t<3, double> Phi(inputBox);
fptr(&rho, &Phi, 1);
```

This is a specification dressed as a program

- Needs to be clean and concise
- No code level optimizations and tricks
- Don’t think “performance” but “correctness”
- *For production code and software engineering*

FFTX Backend: SPIRAL



Executable

Other C/C++ Code

FFTX call site
`fftx_plan(…)`
`fftx_execute(…)`

FFTX call site
`fftx_plan(…)`
`fftx_execute(…)`

FFTX powered by SPIRAL

Paradigm Plug-In: *GPU*

Platform/ISA Plug-In: *CUDA*

Paradigm Plug-In: *Shared memory*

Platform/ISA Plug-In: *OpenMP*

Paradigm Plug-In: *SIMD instructions*

Platform/ISA Plug-In: *AVX, AVX2*

SPIRAL module:
Code synthesis, trade-offs reconfiguration, statistics

Code module 1
Pruned FFT
OpenMP + AVX2

Code module 2
I/O Pruned Convolution
CUDA

Extensible platform and programming model definitions

Core system: SPIRAL engine

Automatically generated tuned components and special cases

C/C++ FFTX Program Trace

```
fftx_session := [
  rec(op := "fftx_init", flags := IntHexString("8000000")),
  rec(op := "fftx_create_data_real", rank := 1, dims := [ rec(n := 4, is := 1, os := 1) ],
    ptr := IntHexString("000000D236DD2460")),
  ...
  rec(op := "fftx_create_zero_temp_real", rank := 1,
    dim
  rec(o                                                                                    1,
    dat
      i
      o
  rec(o
    ran
    how
    inp
    fla
  rec(o
    ran
    ds
    dof
    inp
    dat
    callback := [
      rec(op := "call", inp := IntHexString("A000000000000001"),
        outp := IntHexString("A000000000000002"), data := IntHexString("A000000000000003")),
      rec(op := "FFTX_COMPLEX_VAR", var := IntHexString("000000D236A0FA30"),
        re := 0.000000e+00, im := 0.000000e+00),
      rec(op := "FFTX_COMPLEX_MOV", target := IntHexString("000000D236A0FA30"),
        source := IntHexString("A000000000000001")),
      rec(op := "FFTX_COMPLEX_MUL", target := IntHexString("000000D236A0FA30"),
        source := IntHexString("A000000000000003")),
      ...
```

The whole convolution kernel is captured

- DAG with all dependencies
- User-defined call-backs
- Captures prunining, zero-padding and symmetries
- *Lifts sequence of C++ library calls to a specification*

SPIRAL Script Captures Performance Engineering

```
# Pruned 3D Real Convolution Pattern
Import(realdft);
Import(filtering);

# set up algorithms needed for multi-dimensional pruned real convolution
```

Recognizes pattern and applies code generation

- Developed by performance engineer + application specialist
- Casts FFTX call sequence as SPIRAL non-terminal
- Does code generation and autotuning
- *Clear separation of concerns frontend/backend*

```
sym := var.fresh_t("S", TArray(TReal, 2*n_freq));
t := IOPrunedRConv(N, sym, 1, [minout..N-1], 1, [0..maxin], true);

# generate code and autotune
rt := DP(t, opts)[1].ruletree;
c := CodeRuleTree(rt, opts);

# create files
PrintTo(name::".c", PrintCode(name, c, opts));
```

Backend: SPIRAL Code Generation

```
__global__ void ker_code0(int *D48, double *D49, double *D50, double *D51, int *D52, double  *X) {
    __shared__ double T235[260];
    ...
    if (((threadIdx.x < 13))) {
        for(int i96 = 0; i96 <= 4; i96++) {
            int a31, a32, a33, a34;
            a31 = (2*i96);
            a32 = (threadIdx.x + (13*a31));
            a33 = (threadIdx.x + (13*((a31 + 5) % 10)));
            a34 = (4*i96);
            *((((T235 + 0) + a34) + (20*threadIdx.x))) = (*((T6 + a32)) + *((T6 + a33)));
            *(((1 + (T235 + 0) + a34) + (20*threadIdx.x))) = 0.0;
            *(((2 + (T235 + 0) + a34) + (20*threadIdx.x))) = (*((T6 + a32)) - *((T6 + a33)));
            *(((3 + (T235 + 0) + a34) + (20*threadIdx.x))) = 0.0;
        }
        double t261, t262, t263, t264, t265, t266, t267, t268;
        int a129;
        t263 = (*((((T235+0)+12)+(20*threadIdx.x)))+*((((T235+0)+8)+(20*threadIdx.x))));
        t264 = (*((((T235+0)+12)+(20*threadIdx.x)))-*((((T235+0)+8)+(20*threadIdx.x))));
        ...
        *((3 + T5 + a129)) = ((0.58778525229247314*t268) - (0.95105651629515353*t266));
    }
    __syncwarp();
    if (((threadIdx.x < 1))) {
        double t305, t306, t307, t308, t309, t310, t311, t312, t313, t314, t315, t316;
        int a387;
        t305 = (*((T5 + 12)) + *((T5 + 144)));
        ...
```

FFTX/SPIRAL with CUDA backend

Early result:

130 Gflop/s

on par with cuFFT

3,000 lines of code, kernel fusion, cross call data layout transforms

Outline

- Introduction
- Specifying computation
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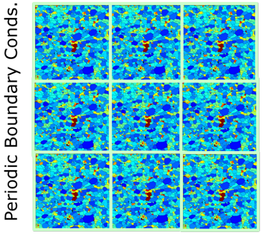
- FFTX: A Library Frontend for SPIRAL
- **Summary**

FFTX Extension For MASSIF/LANL

Convolution with Rank-4 Tensor

MPI vpFFT: Viscoplastic Polycrystals



Rate-sensitive approach
(n = Viscoplastic exponent)

(1) $\dot{\varepsilon}(x) = \dot{\gamma}_o \sum_s m^s(x) \left(\frac{m^s(x) : \sigma'(x)}{\tau_o^s(x)} \right)^n$

Schmid Tensor

Threshold Stress (Hardening of deformed system)

(2)
$$\begin{aligned} \sigma'(x) &= \sigma'(x) + L^o : \dot{\varepsilon}(x) - L^o : \dot{\varepsilon}(x) \\ &= L^o : \dot{\varepsilon}(x) + \left(\sigma'(x) - L^o : \dot{\varepsilon}(x) \right) \\ &= L^o : \dot{\varepsilon}(x) + \tau(x) \end{aligned}$$

Stiffness of a Linear Reference Medium

Fluctuation (Heterogeneity Field)

Function of Solution → Requires Iterative Procedure

(3)
$$\left| \begin{array}{ll} L^o_{ijkl} v_{k,lj}(x) + \tau_{ij,j}(x) - p_{,i}(x) = 0 & \textit{in RVE} \\ v_{k,k}(x) = 0 & \textit{in RVE} \\ \textit{periodic boundary conditions across RVE} & \end{array} \right.$$

Equilibrium + Incompressibility

Slip Geometry : $m_{ij}^{(s)} = b_i^{(s)} n_j^{(s)} = \mathbf{b}^{(s)} \otimes \mathbf{n}^{(s)}$

Green's Function Method, see book by MURA

(4)
$$\left| \begin{array}{l} L^o_{ijkl}\, G_{km,lj}(x-x') - H_{m,i}(x-x') + \delta_{im}\delta(x-x') = 0 \\ G_{km,k}(x-x') = 0 \end{array} \right.$$

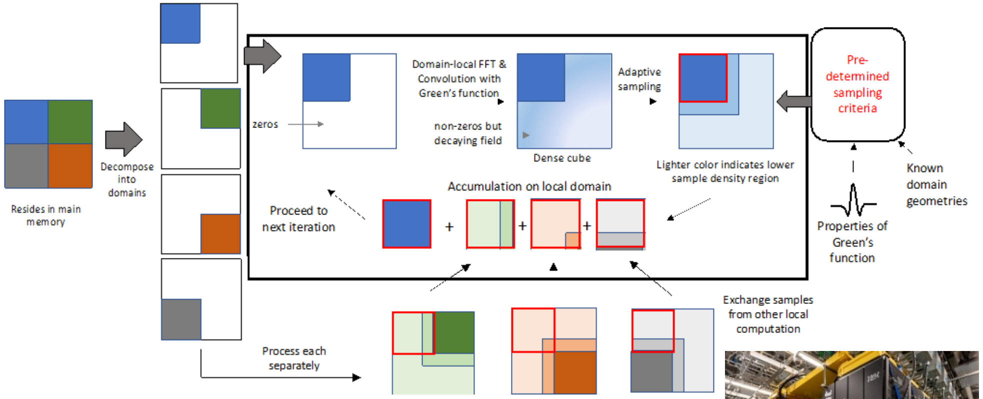
$$\tilde{v}_{i,j}(x) = sym\left(\int_{R^3} G_{ik,jl}(x-x')\ \tau_{kl}(x')\, dx' \right)$$

FFT code MPI parallelized via FFTW

Upon Convergence:
Stress, Strain-Rate and Slip-Rate Fields are obtained
Convergence provides a compromise between

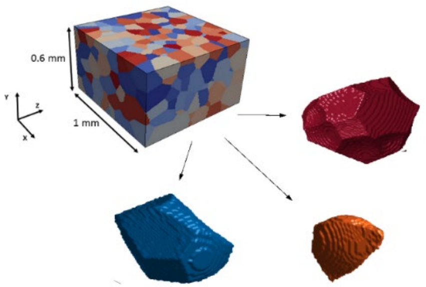
R. Lebensohn (LANL), A.D. Rollett (CMU)

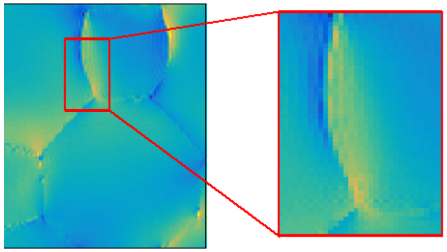
Challenge: Fitting Into GPU Memory


Resides in main memory
Decompose into domains
zeros
Domain-local FFT & Convolution with Green's function
non-zeros but decaying field
Dense cube
Adaptive sampling
Lighter color indicates lower sample density region
Pre-determined sampling criteria
Properties of Green's function
Known domain geometries
Accumulation on local domain
Proceed to next iteration
+
+
+
Process each separately
Exchange samples from other local computation

FORTRAN + MATLAB, reduce MPI traffic towards FFTX/SpectralPACK

Irregular Domain Decomposition





Signal processing + PDE tricks to compress

Performance Model

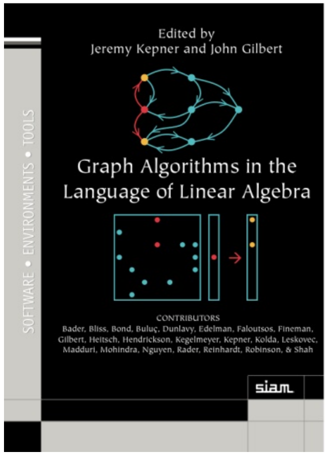
90224.994 32
11301.902 64

Problem size	Domain size	Compute time/domain [s]	No. of domains	Total compute time (single GPU) [Hrs]	Memory for Domain-local FFT [GB]
1024 × 1024 × 1024	128 × 128 × 128	0.292	512	0.041	1
1024 × 1024 × 1024	512 × 512 × 512	0.329	8	0.0007	4
2048 × 2048 × 2048	128 × 128 × 128	2.352	4096	2.676	4
2048 × 2048 × 2048	512 × 512 × 512	2.485	64	0.044	16
4096 × 4096 × 4096	128 × 128 × 128	19.079	32,768	173.662	16
4096 × 4096 × 4096	512 × 512 × 512	19.589	512	2.786	64
4096 × 4096 × 4096	1024 × 1024 × 1024	20.399	64	0.362	128
8192 × 8192 × 8192	64 × 64 × 64	154.882	2,097,152	90224.994	32
8192 × 8192 × 8192	128 × 128 × 128	155.208	2,62,144	11301.902	64
8192 × 8192 × 8192	512 × 512 × 512	157.272	4096	178.940	256
8192 × 8192 × 8192	1024 × 1024 × 1024	160.303	512	22.798	512

Model: 8k x 8k x 8k possible on Summit

Graph Algorithms in SPIRAL

Foundation

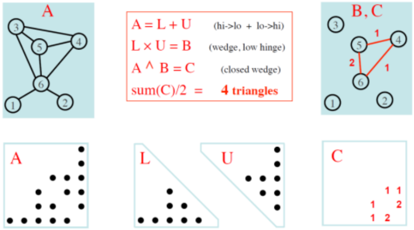

Edited by
Jeremy Kepner and John Gilbert

Graph Algorithms in the Language of Linear Algebra

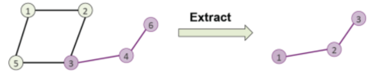
SOFTWARE • ENVIRONMENTS • TOOLS

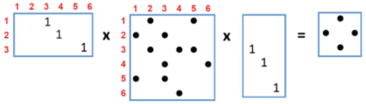
CONTRIBUTORS
Bader, Bliss, Bond, Buluç, Dunlavy, Edelman, Faloutsos, Fineman, Gilbert, Heitsch, Hendrickson, Kegelmeyer, Kepner, Kolda, Leskovec, Madduri, Mohindra, Nguyen, Rader, Reinhardt, Robinson, & Shah

siam



A = L + U (hi->lo + lo->hi)
L × U = B (wedge, low hinge)
A ^ B = C (closed wedge)
sum(C)/2 = 4 triangles





Triangle Counting in SPIRAL

```
TriangleCount()
```

$$\Delta = \Delta + \tfrac{1}{2}\alpha_{10}A_{00}\alpha_{01}$$

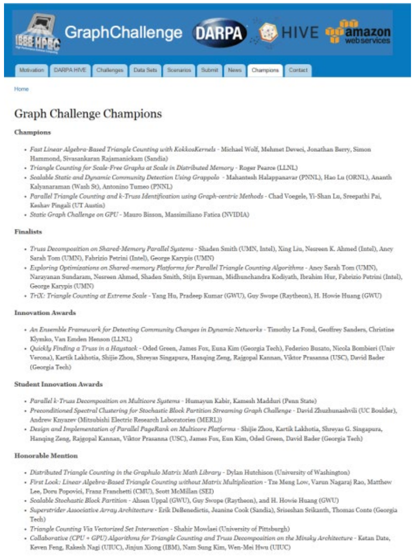
```
BB(
  Accum(i4, 1, X.N-1,
    Accum_X(i6, [ i4, 0 ], i4,
      Dot([ i6, add(i4, V(1)) ],
          [ i4, add(i4, V(1)) ],
           sub(sub(X.N, i4), V(1))
)))
```

Formalization

Operation	Mathematical Description	Output	Inputs
mxm	$C\langle\neg M, z\rangle = C \odot (A^T \oplus.\otimes B^T)$	C	¬, M, z, ⊙, A, T, ⊕.⊗, B, T
mxv, (vxm)	$c\langle\neg m, z\rangle = c \odot (A^T \oplus.\otimes b)$	c	¬, m, z, ⊙, A, T, ⊕.⊗, b
eWiseMult	$C\langle\neg M, z\rangle = C \odot (A^T \otimes B^T)$	C	¬, M, z, ⊙, A, T, ⊗, B, T
eWiseAdd	$C\langle\neg M, z\rangle = C \odot (A^T \oplus B^T)$	C	¬, M, z, ⊙, A, T, ⊕, B, T
reduce (row)	$c\langle\neg m, z\rangle = c \odot [\oplus_j A^T(:,j)]$	c	¬, m, z, ⊙, A, T, ⊕
apply	$C\langle\neg M, z\rangle = C \odot f(A^T)$	C	¬, M, z, ⊙, A, T, *f*
transpose	$C\langle\neg M, z\rangle = C \odot A^T$	C	¬, M, z, ⊙, A (T)
extract	$C\langle\neg M, z\rangle = C \odot A^T(i,j)$	C	¬, M, z, ⊙, A, T, i, j
assign	$C\langle\neg M, z\rangle (i,j) = C(i,j) \odot A^T$	C	¬, M, z, ⊙, A, T, i, j
build (meth.)	$C = \mathbb{S}^{mxn}(i,j,v,\odot)$	C	⊙, m, n, i, j, v
extractTuples (meth.)	$(i,j,v) = A$	i,j,v	A

Notation: **i,j** – index arrays, **v** – scalar array, **m** – 1D mask, **other bold-lower** – vector (column), **M** – 2D mask, **other bold-caps** – matrix, **T** – transpose, ¬ - structural complement, z – clear output, ⊕ monoid/binary function, ⊕.⊗ semiring, blue – optional parameters, red – optional modifiers

HPEC Graph Challenge



Graph Challenge Champions

Champions

- *Fast Linear Algebra-Based Triangle Counting with KokkosKernels* - Michael Wolf, Mehmet Deveci, Jonathan Berry, Simon Hammond, Sivasankaran Rajamanickam (Sandia)
- *Triangle Counting for Scale-Free Graphs at Scale in Distributed Memory* - Roger Pearce (LLNL)
- *Scalable Static and Dynamic Community Detection Using Grappolo* - Mahantesh Halappanavar (PNNL), Hao Lu (ORNL), Ananth Kalyanaraman (Wash St), Antonino Tumeo (PNNL)
- *Parallel Triangle Counting and k-Truss Identification using Graph-centric Methods* - Chad Voegele, Yi-Shan Lu, Sreepathi Pai, Keshav Pingali (UT Austin)
- *Static Graph Challenge on GPU* - Mauro Bisson, Massimiliano Fatica (NVIDIA)

Finalists

- *Truss Decomposition on Shared-Memory Parallel Systems* - Shaden Smith (UMN, Intel), Xing Liu, Nesreen K. Ahmed (Intel), Ancy Sarah Tom (UMN), Fabrizio Petrini (Intel), George Karypis (UMN)
- *Exploring Optimizations on Shared-memory Platforms for Parallel Triangle Counting Algorithms* - Ancy Sarah Tom (UMN), Narayanan Sundaram, Nesreen Ahmed, Shaden Smith, Stijn Eyerman, Midhunchandra Kodiyath, Ibrahim Hur, Fabrizio Petrini (Intel), George Karypis (UMN)
- *TriX: Triangle Counting at Extreme Scale* - Yang Hu, Pradeep Kumar (GWU), Guy Swope (Raytheon), H. Howie Huang (GWU)

Innovation Awards

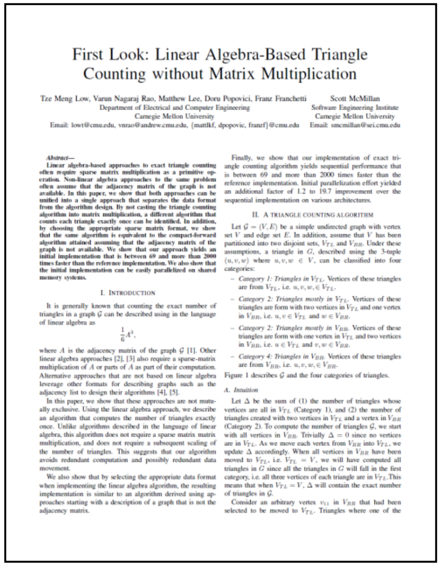
- *An Ensemble Framework for Detecting Community Changes in Dynamic Networks* - Timothy La Fond, Geoffrey Sanders, Christine Klymko, Van Emden Henson (LLNL)
- *Quickly Finding a Truss in a Haystack* - Oded Green, James Fox, Euna Kim (Georgia Tech), Federico Busato, Nicola Bombieri (Univ Verona), Kartik Lakhotia, Shijie Zhou, Shreyas Singapura, Hanqing Zeng, Rajgopal Kannan, Viktor Prasanna (USC), David Bader (Georgia Tech)

Student Innovation Awards

- *Parallel k-Truss Decomposition on Multicore Systems* - Humayun Kabir, Kamesh Madduri (Penn State)
- *Preconditioned Spectral Clustering for Stochastic Block Partition Streaming Graph Challenge* - David Zhuzhunashvili (UC Boulder), Andrew Knyazev (Mitsubishi Electric Research Laboratories (MERL))
- *Design and Implementation of Parallel PageRank on Multicore Platforms* - Shijie Zhou, Kartik Lakhotia, Shreyas G. Singapura, Hanqing Zeng, Rajgopal Kannan, Viktor Prasanna (USC), James Fox, Eun Kim, Oded Green, David Bader (Georgia Tech)

Honorable Mention

- *Distributed Triangle Counting in the Graphulo Matrix Math Library* - Dylan Hutchison (University of Washington)
- *First Look: Linear Algebra-Based Triangle Counting without Matrix Multiplication* - Tze Meng Low, Varun Nagaraj Rao, Matthew Lee, Doru Popovici, Franz Franchetti (CMU), Scott McMillan (SEI)
- *Scalable Stochastic Block Partition* - Ahsen Uppal (GWU), Guy Swope (Raytheon), and H. Howie Huang (GWU)
- *Superstrider Associative Array Architecture* - Erik DeBenedictis, Jeanine Cook (Sandia), Sriseshan Srikanth, Thomas Conte (Georgia Tech)
- *Triangle Counting Via Vectorized Set Intersection* - Shahir Mowlaei (University of Pittsburgh)
- *Collaborative (CPU + GPU) Algorithms for Triangle Counting and Truss Decomposition on the Minsky Architecture* - Ketan Date, Keven Feng, Rakesh Nagi (UIUC), Jinjun Xiong (IBM), Nam Sung Kim, Wen-Mei Hwu (UIUC)



First Look: Linear Algebra-Based Triangle Counting without Matrix Multiplication

Tze Meng Low, Varun Nagaraj Rao, Matthew Lee, Doru Popovici, Franz Franchetti
Department of Electrical and Computer Engineering
Carnegie Mellon University
Email: lowt@cmu.edu, vnrao@andrew.cmu.edu, {mattlkf, dpopovic, franzf}@cmu.edu

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Software Engineering Institute
Carnegie Mellon University
Email: smcmillan@sei.cmu.edu

***Abstract*—Linear algebra-based approaches to exact triangle counting often require sparse matrix multiplication as a primitive operation. Non-linear algebra approaches to the same problem often assume that the adjacency matrix of the graph is not available. In this paper, we show that both approaches can be unified into a single approach that separates the data format from the algorithm design. By not casting the triangle counting algorithm into matrix multiplication, a different algorithm that counts each triangle exactly once can be identified. In addition, by choosing the appropriate sparse matrix format, we show that the same algorithm is equivalent to the compact-forward algorithm attained assuming that the adjacency matrix of the graph is not available. We show that our approach yields an initial implementation that is between 69 and more than 2000 times faster than the reference implementation. We also show that the initial implementation can be easily parallelized on shared memory systems.**

I. Introduction

It is generally known that counting the exact number of triangles in a graph $\mathcal{G}$ can be described using in the language of linear algebra as

$$\frac{1}{6}A^3,$$

where A is the adjacency matrix of the graph $\mathcal{G}$ [1]. Other linear algebra approaches [2], [3] also require a sparse-matrix multiplication of A or parts of A as part of their computation. Alternative approaches that are not based on linear algebra leverage other formats for describing graphs such as the adjacency list to design their algorithms [4], [5].

In this paper, we show that these approaches are not mutually exclusive. Using the linear algebra approach, we describe an algorithm that computes the number of triangles exactly once. Unlike algorithms described in the language of linear algebra, this algorithm does not require a sparse matrix matrix multiplication, and does not require a subsequent scaling of the number of triangles. This suggests that our algorithm avoids redundant computation and possibly redundant data movement.

We also show that by selecting the appropriate data format when implementing the linear algebra algorithm, the resulting implementation is similar to an algorithm derived using approaches starting with a description of a graph that is not the adjacency matrix.

Finally, we show that our implementation of exact triangle counting algorithm yields sequential performance that is between 69 and more than 2000 times faster than the reference implementation. Initial parallelization effort yielded an additional factor of 1.2 to 19.7 improvement over the sequential implementation on various architectures.

II. A Triangle Counting Algorithm

Let $\mathcal{G} = (V, E)$ be a simple undirected graph with vertex set V and edge set E. In addition, assume that V has been partitioned into two disjoint sets, V_{TL} and V_{BR}. Under these assumptions, a triangle in G, described using the 3-tuple (u, v, w) where $u, v, w \in V$, can be classified into four categories:

- *Category 1: Triangles in V_{TL}.* Vertices of these triangles are from V_{TL}, i.e. $u, v, w, \in V_{TL}$.
- *Category 2: Triangles mostly in V_{TL}.* Vertices of these triangles are form with two vertices in V_{TL} and one vertex in V_{BR}, i.e. $u, v \in V_{TL}$ and $w \in V_{BR}$.
- *Category 2: Triangles mostly in V_{BR}.* Vertices of these triangles are form with one vertex in V_{TL} and two vertices in V_{BR}, i.e. $u \in V_{TL}$ and $v, w \in V_{BR}$.
- *Category 4: Triangles in V_{BR}.* Vertices of these triangles are from V_{BR}, i.e. $u, v, w, \in V_{BR}$.

Figure 1 describes $\mathcal{G}$ and the four categories of triangles.

A. Intuition

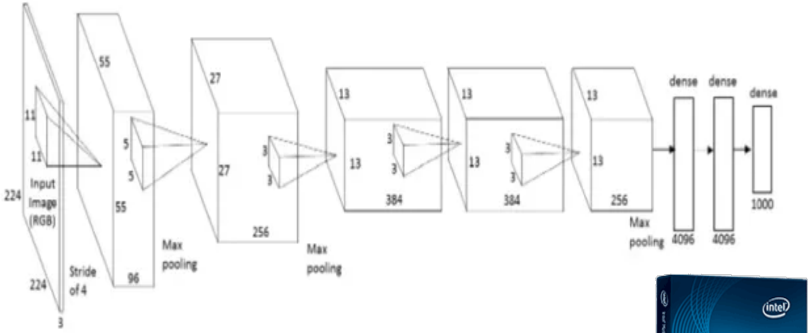
Let Δ be the sum of (1) the number of triangles whose vertices are all in V_{TL} (Category 1), and (2) the number of triangles created with two vertices in V_{TL} and a vertex in V_{BR} (Category 2). To compute the number of triangles $\mathcal{G}$, we start with all vertices in V_{BR}. Trivially $\Delta = 0$ since no vertices are in V_{TL}. As we move each vertex from V_{BR} into V_{TL}, we update Δ accordingly. When all vertices in V_{BR} have been moved to V_{TL}, i.e. $V_{TL} = V$, we will have computed all triangles in G since all the triangles in G will fall in the first category, i.e. all three vertices of each triangle are in V_{TL}.This means that when $V_{TL} = V$, Δ will contain the exact number of triangles in $\mathcal{G}$.

Consider an arbitrary vertex v_{11} in V_{BR} that had been selected to be moved to V_{TL}. Triangles where one of the

In collaboration with CMU-SEI

Towards Deep Learning in SPIRAL

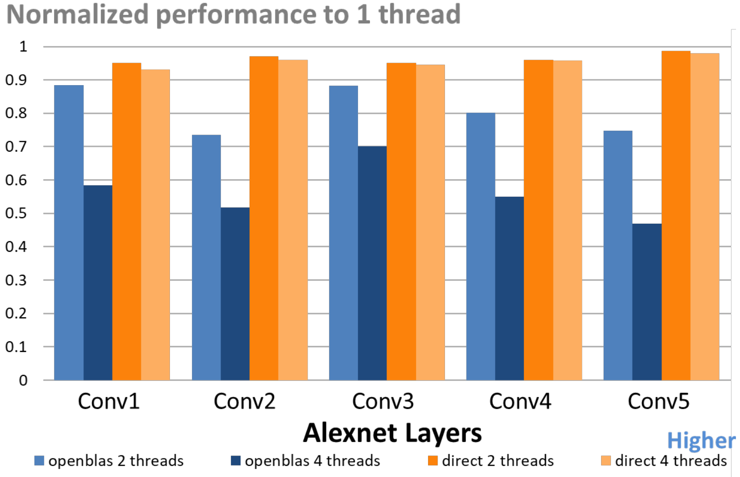
Standard: Use GEMM



```
dgemm(transa, transb, m, n, k, alpha,
      a, lda, b, ldb, beta, c, ldc)
```

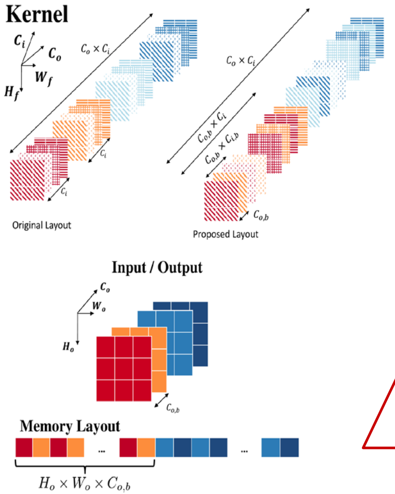
Direct CNN – More efficient

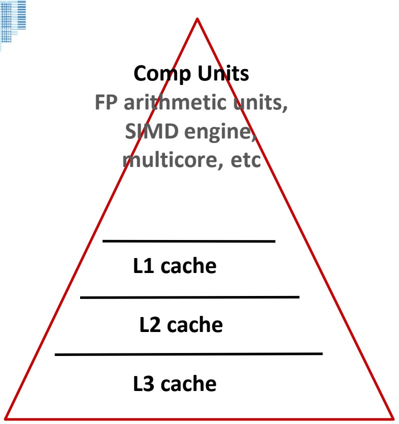
Scalability on AMD Piledriver



Jiyuan Zhang, Franz Franchetti, Tze Meng Low, "High Performance Zero-Memory Overhead Direct Convolutions", 2018, International Conference of Machine Learning

CNN/System Friendly Layout





Comp Units
FP arithmetic units,
SIMD engine,
multicore, etc

L1 cache

L2 cache

L3 cache

Towards CNNs in SPIRAL

- Level 0: simple C program
  implements the algorithm cleanly
- Level 1: C macros plus search script
  use C preprocessor for meta-programming
- Level 2: scripting for code specialization
  text-based program generation, e.g., ATLAS
- Level 3: add compiler technology
  internal code representation, e.g., FFTW's genfft
- Level 4: synthesize the program from scratch
  high level representation, e.g., TCE and Spiral

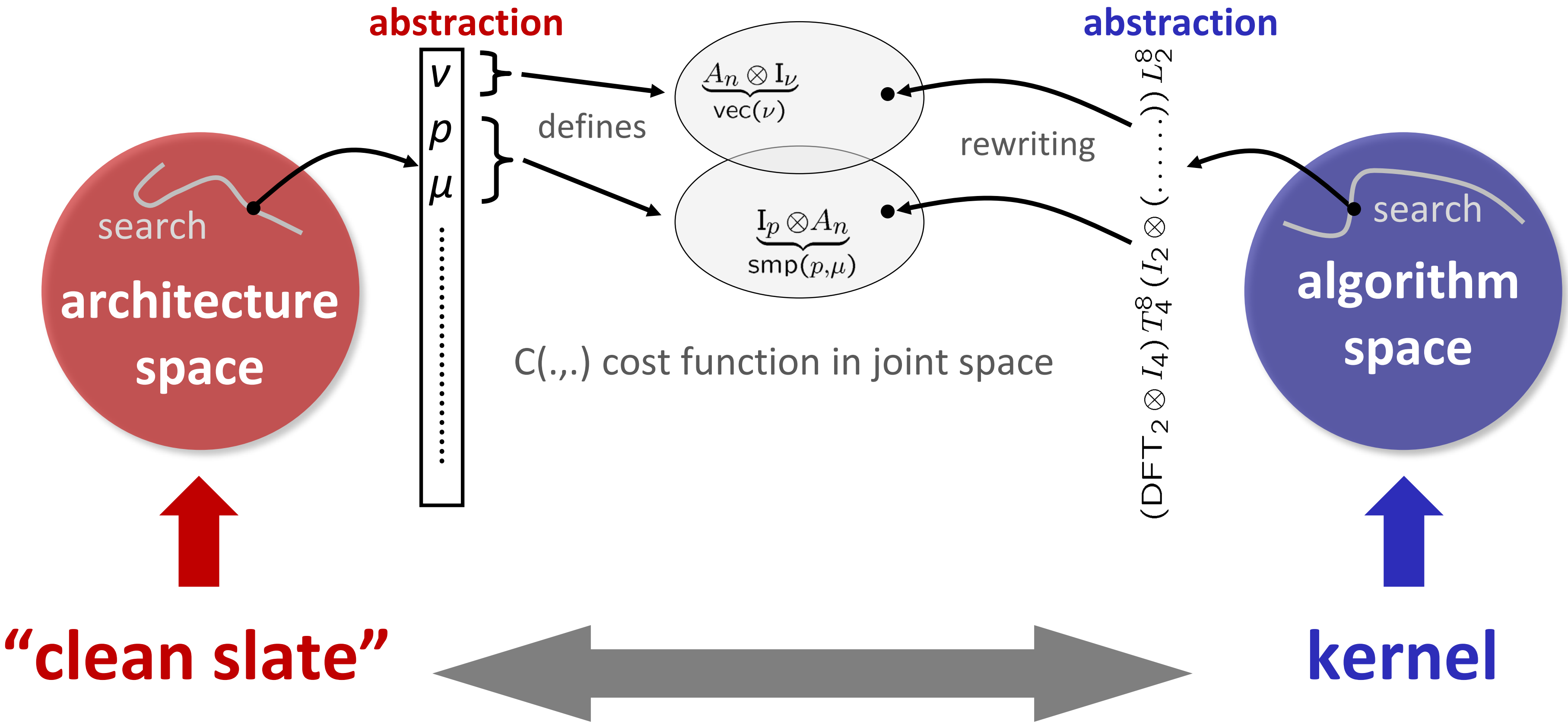
Higher

Co-Optimizing Architecture and Kernel

Architectural parameter:
Vector length,
#processors, …

Model: common abstraction
= spaces of matching formulas

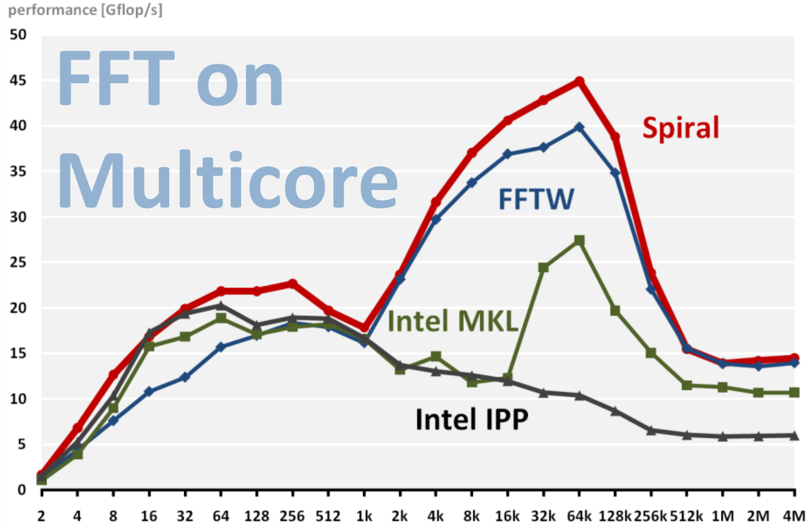
Kernel:
problem size,
algorithm choice


abstraction
abstraction
ν
p
μ
defines
$\underbrace{A_n \otimes \mathrm{I}_\nu}_{\mathrm{vec}(\nu)}$
$\underbrace{\mathrm{I}_p \otimes A_n}_{\mathrm{smp}(p,\mu)}$
rewriting
$(\mathrm{DFT}_2 \otimes I_4)\, T^8_4\, (I_2 \otimes (\ldots\ldots))\, L^8_2$
search
architecture space
search
algorithm space
C(.,.) cost function in joint space
"clean slate"
kernel

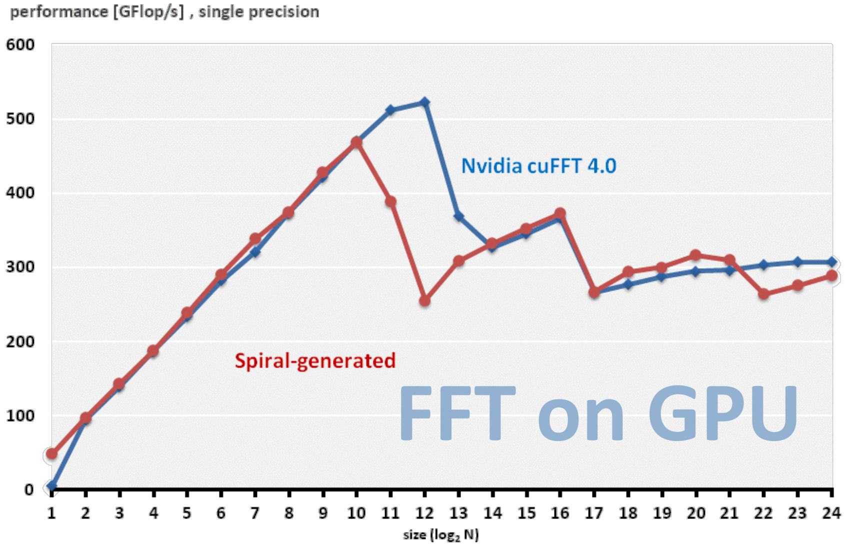
Goal: SPIRAL co-designed RISC-V accelerator chip, taped out

Some Results: FFTs and Spectral Algorithms

1D DFT on 3.3 GHz Sandy Bridge (4 Cores, AVX)

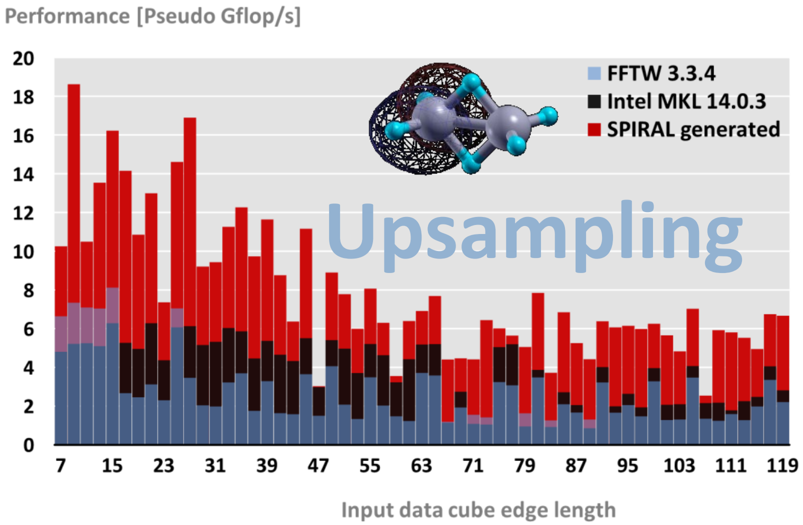


1D Batch DFT (Nvidia GTX 480)

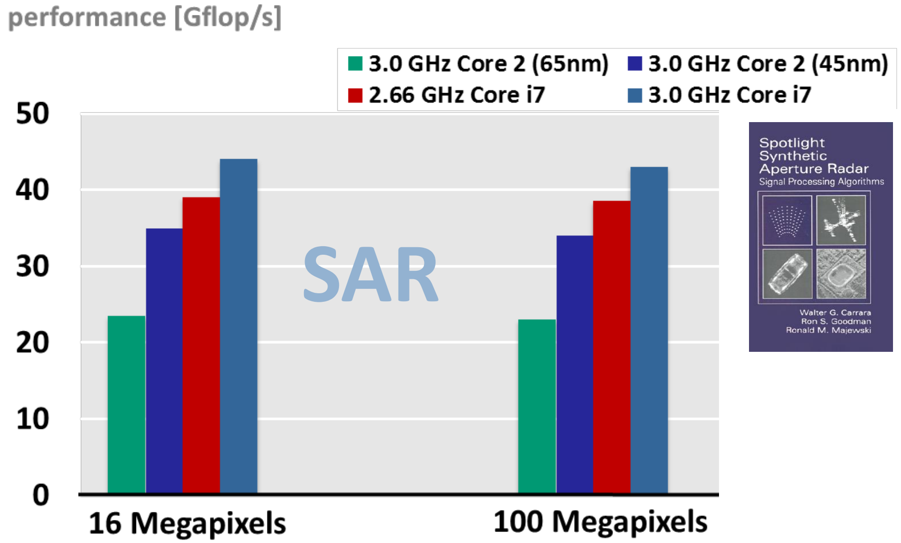


Performance of 2x2x2 Upsampling on Haswell

3.5 GHz, AVX, double precision, interleaved input, single core



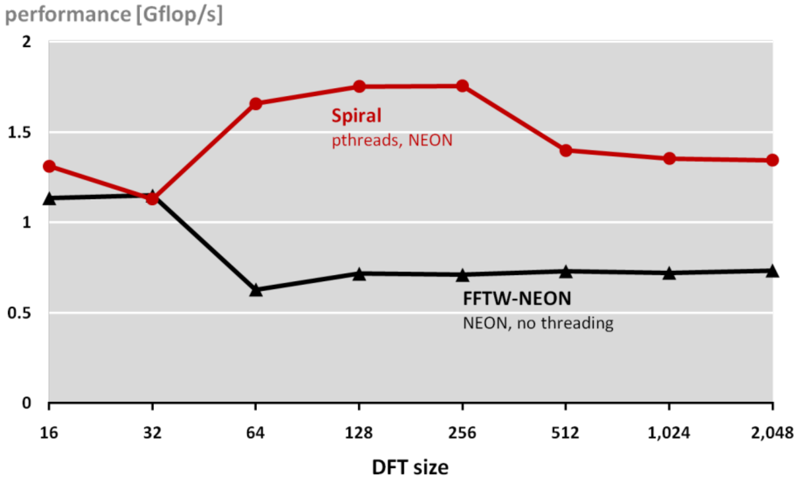
PFA SAR Image Formation on Intel platforms



From Cell Phone To Supercomputer

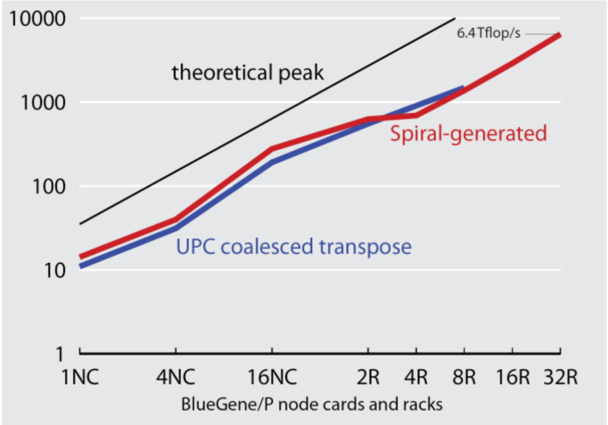
DFT on Samsung Galaxy S II

Dual-core 1.2 GHz Cortex-A9 with NEON ISA



Global FFT (1D FFT, HPC Challenge)

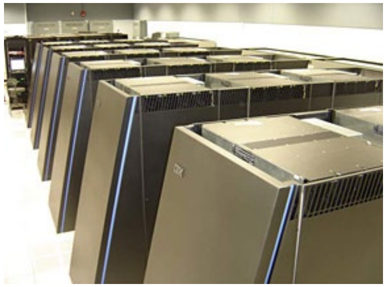
performance [Gflop/s]



6.4 Tflop/s on BlueGene/P

Samsung i9100 Galaxy S II
Dual-core ARM at 1.2GHz with NEON ISA

BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz



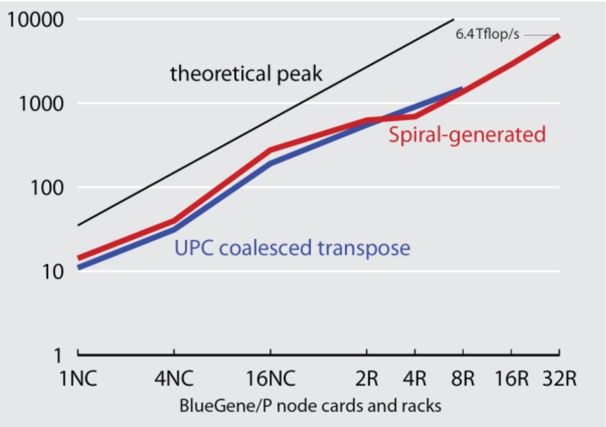
F. Gygi, E. W. Draeger, M. Schulz, B. R. de Supinski, J. A. Gunnels, V. Austel, J. C. Sexton, F. Franchetti, S. Kral, C. W. Ueberhuber, J. Lorenz, **“Large-Scale Electronic Structure Calculations of High-Z Metals on the BlueGene/L Platform,”** In Proceedings of Supercomputing, 2006. ***2006 Gordon Bell Prize (Peak Performance Award).***

G. Almási, B. Dalton, L. L. Hu, F. Franchetti, Y. Liu, A. Sidelnik, T. Spelce, I. G. Tānase, E. Tiotto, Y. Voronenko, X. Xue, **“2010 IBM HPC Challenge Class II Submission,”** ***2010 HPC Challenge Class II Award (Most Productive System).***

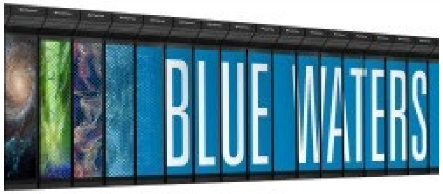
SPIRAL: Success in HPC/Supercomputing

- **NCSA Blue Waters**
  PAID Program, FFTs for Blue Waters
- **RIKEN K computer**
  FFTs for the HPC-ACE ISA
- **LANL RoadRunner**
  FFTs for the Cell processor
- **PSC/XSEDE Bridges**
  Large size FFTs
- **LLNL BlueGene/L and P**
  FFTW for BlueGene/L's Double FPU
- **ANL BlueGene/Q Mira**
  Early Science Program, FFTW for BGQ QPX

Global FFT (1D FFT, HPC Challenge)
performance [Gflop/s]



BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz

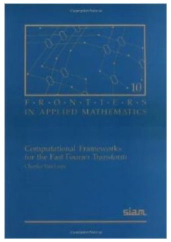


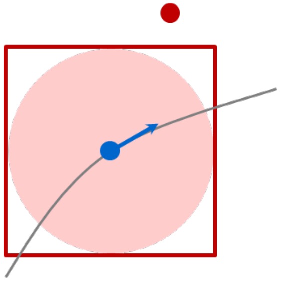
2006 Gordon Bell Prize (Peak Performance Award) with LLNL and IBM

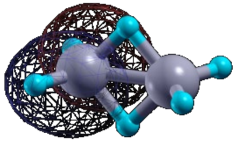
2010 HPC Challenge Class II Award (Most Productive System) with ANL and IBM

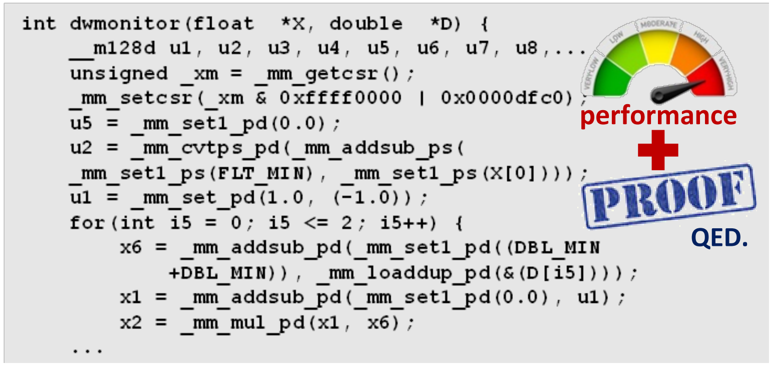
SPIRAL: AI for High Performance Code

Algorithms


FRONTIERS IN APPLIED MATHEMATICS
10
Computational Frameworks for the Fast Fourier Transform
Charles Van Loan
siam

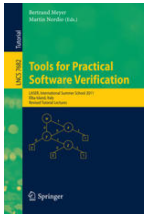


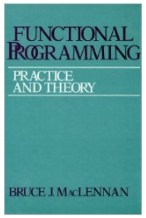


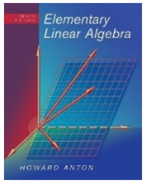


```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8,...
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(
    _mm_set1_ps(FLT_MIN), _mm_set1_ps(X[0])));
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN
            +DBL_MIN)), _mm_loaddup_pd(&(D[i5])));
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
    ...
```

Correctness





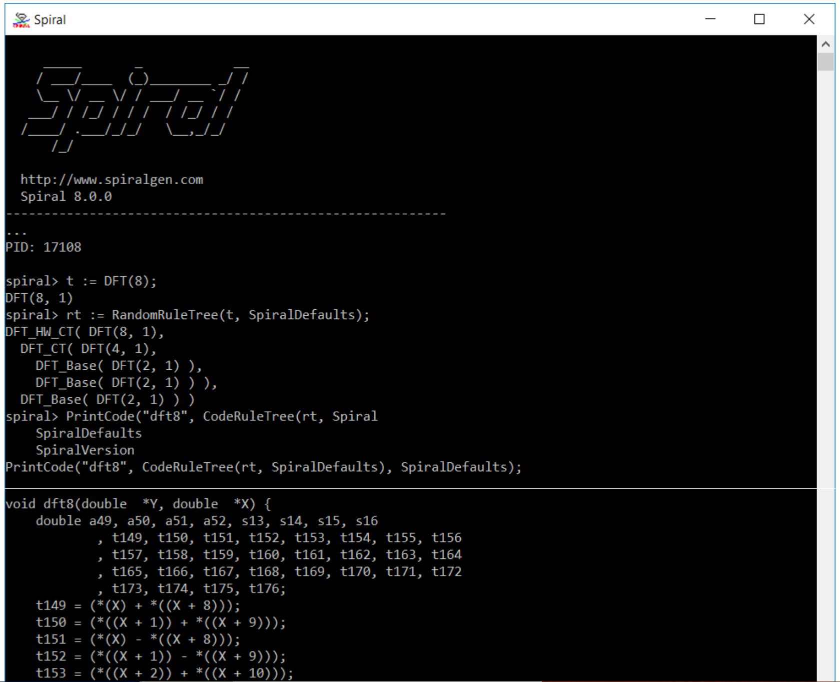


Hardware

SPIRAL 8.1.0: Available Under Open Source

- **Open Source SPIRAL available**
  - non-viral license (BSD)
  - Initial version, effort ongoing to open source whole system
  - Commercial support via SpiralGen, Inc.
- **Developed over 20 years**
  - Funding: DARPA (OPAL, DESA, HACMS, PERFECT, BRASS), NSF, ONR, DoD HPC, JPL, DOE, CMU SEI, Intel, Nvidia, Mercury
- **Open sourced under DARPA PERFECT, continuing under DOE ECP**
- **Tutorial material available online**

www.spiral.net



```
   ____        _           __
  / ___/____  (_)________ _/ /
  \__ \/ __ \/ / ___/ __ `/ /
 ___/ / /_/ / / /  / /_/ / /
/____/ .___/_/_/   \__,_/_/
    /_/

  http://www.spiralgen.com
  Spiral 8.0.0
----------------------------------------------------------
...
PID: 17108

spiral> t := DFT(8);
DFT(8, 1)
spiral> rt := RandomRuleTree(t, SpiralDefaults);
DFT_HW_CT( DFT(8, 1),
  DFT_CT( DFT(4, 1),
    DFT_Base( DFT(2, 1) ),
    DFT_Base( DFT(2, 1) ) ),
  DFT_Base( DFT(2, 1) ) )
spiral> PrintCode("dft8", CodeRuleTree(rt, Spiral
    SpiralDefaults
    SpiralVersion
PrintCode("dft8", CodeRuleTree(rt, SpiralDefaults), SpiralDefaults);

void dft8(double  *Y, double  *X) {
    double a49, a50, a51, a52, s13, s14, s15, s16
            , t149, t150, t151, t152, t153, t154, t155, t156
            , t157, t158, t159, t160, t161, t162, t163, t164
            , t165, t166, t167, t168, t169, t170, t171, t172
            , t173, t174, t175, t176;
    t149 = (*(X) + *((X + 8)));
    t150 = (*((X + 1)) + *((X + 9)));
    t151 = (*(X) - *((X + 8)));
    t152 = (*((X + 1)) - *((X + 9)));
    t153 = (*((X + 2)) + *((X + 10)));
```

David Padua
Editor-in-Chief
Encyclopedia of Parallel Computing
Springer
November 2018 | Volume 106 | Number 11
Proceedings of the IEEE
SPECIAL ISSUE
From High-Level Specification to High-Performance Code
IEEE
PROCEEDINGS OF THE IEEE
Special Issue on:
PROGRAM GENERATION, OPTIMIZATION, AND PLATFORM ADAPTATION
IEEE

F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura: SPIRAL: Extreme Performance Portability, **Proceedings of the IEEE, Vol. 106, No. 11, 2018.**
Special Issue on *From High Level Specification to High Performance Code*