SPIRAL:
AI for High Performance Code

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Joint work with the SPIRAL team

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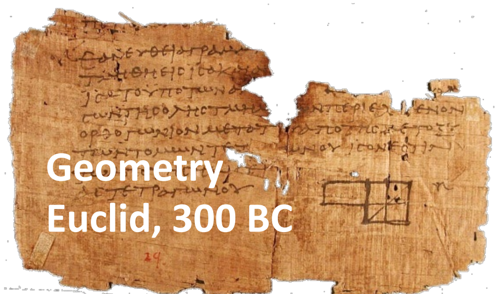
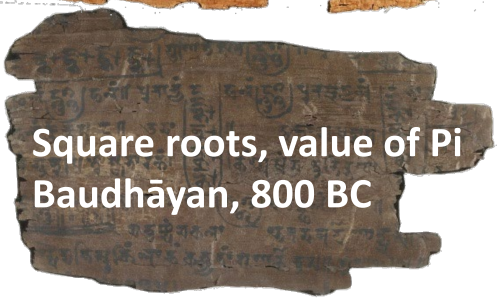
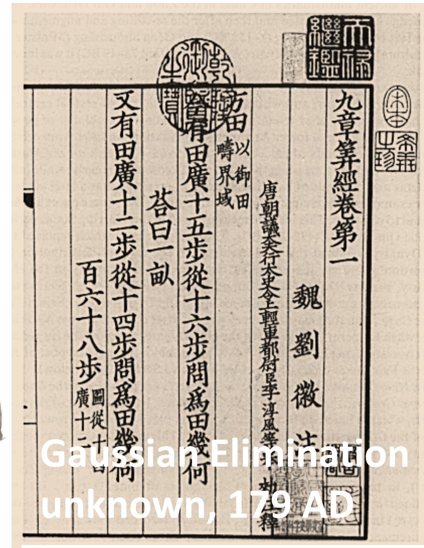
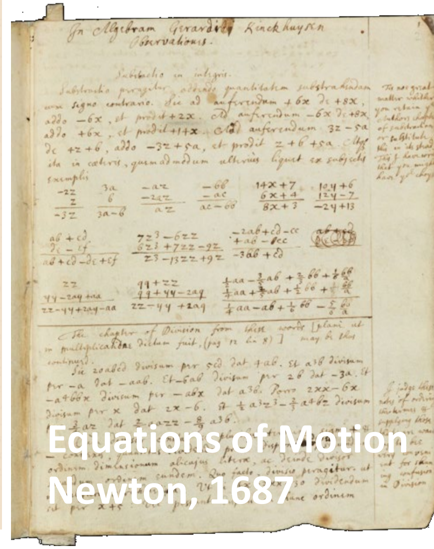
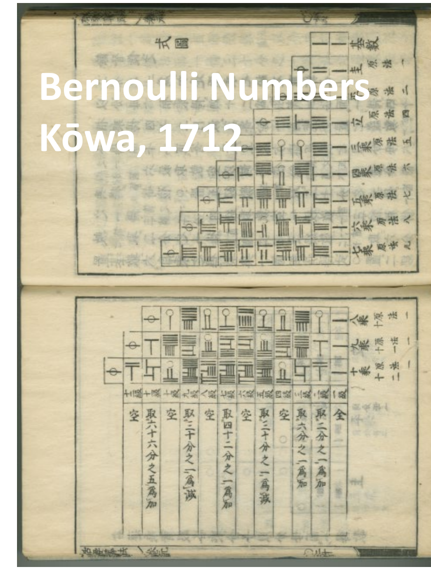
Algorithms and Mathematics: 2,500+ Years



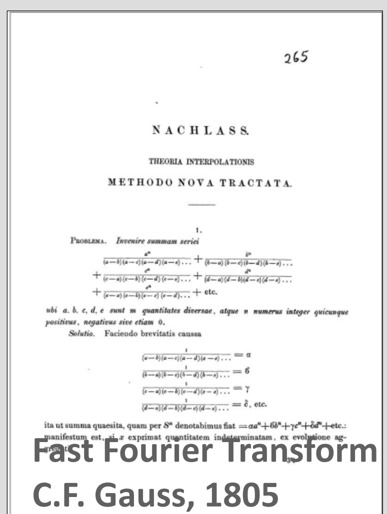








Fast Fourier Transform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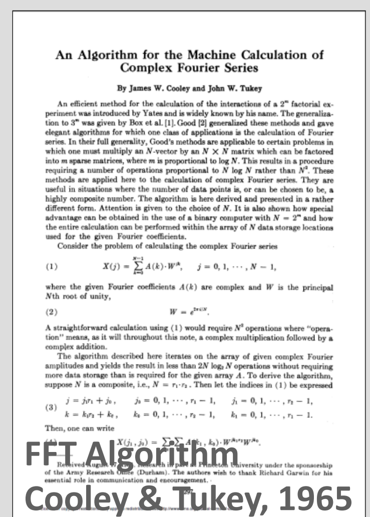
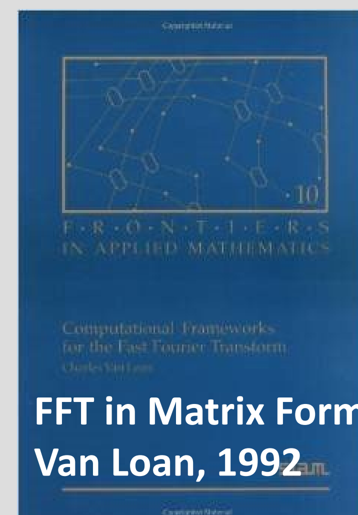
NACHLASS.

THEORIA INTERPOLATIONIS
METHODO NOVA TRACTATA.


An Algorithm for the Machine Calculation of Complex Fourier Series

By James W. Cooley and John W. Tukey



Computing Platforms Over The Years

F-16A/B, C/D, E/F, IN, IQ, N, V: Flying since 1974

Compare: Desktop/workstation class CPUs/machines

Assembly code compatible !!

x86 binary compatible, but 500x parallelism ?!

1972	1989	1994	2006	2011	2018
Intel 8008	IBM PC/XT compatible	IBM RS/6000-390	GeForce 8800	Xeon Phi	Xeon Platinum 8180M
0.2—0.8 MHz	8088 @ 8 MHz, 640kB RAM	256 MB RAM, 6GB HDD	1.3 GHz, 128 shaders	1.3 GHz, 60 cores	28 cores, 2.5-3.6 GHz
Intelligent terminal	360 kB FDD, 720x348 mono	67 MHz Power2+, AIX	16-way SIMT	8/16-way SIMD	2/4/8/16-way SIMD

10^7 – 10^8 compounded performance gain over 45 years

Programming/Languages Libraries Timeline

Popular performance programming languages

- **1953:** Fortran
- **1973:** C
- **1985:** C++
- **1997:** OpenMP
- **2007:** CUDA
- **2009:** OpenCL

Popular performance libraries

- **1979:** BLAS
- **1992:** LAPACK
- **1994:** MPI
- **1995:** ScaLAPACK
- **1995:** PETSc
- **1997:** FFTW

Popular productivity/scripting languages

- **1987:** Perl
- **1989:** Python
- **1993:** Ruby
- **1995:** Java
- **2000:** C#

2019: What $1M Can Buy You

Dell PowerEdge R940
4.5 Tflop/s, 6 TB, 850 W
4x 28 cores, 2.5 GHz

OSS FSAn-4
200 TB PCIe NVMe flash
80 GB/s throughput

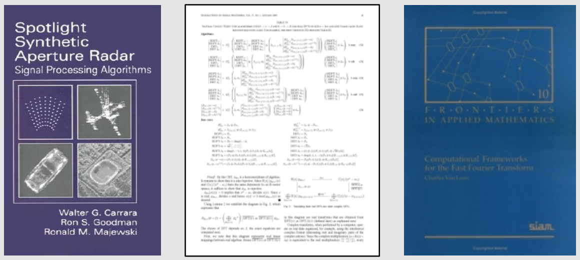
BittWare TeraBox
18M logic elements, 4.9 Tb/sec I/O
8 FPGA cards/16 FPGAs, 2 TB DDR4

AberSAN ZXP4
90x 12TB HDD, 1 kW
1PB raw

Nvidia DGX-1
8x Tesla V100, 3.2 kW
170 Tflop/s, 128 GB

SPIRAL: AI for High Performance Code

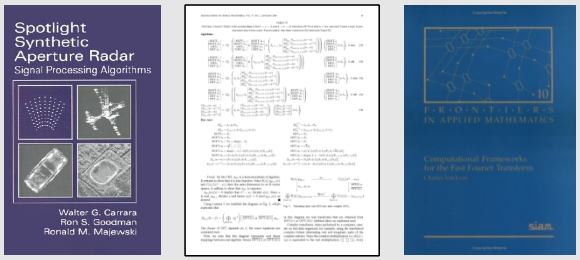
Traditionally



High performance library optimized for given platform

Comparable performance

Spiral Approach



Spiral

High performance library optimized for given platform

Outline

- Introduction
- **Operator Language**
- Achieving Performance Portability
- FFTX: A Library Frontend for SPIRAL
- Summary

OL Operators

Definition

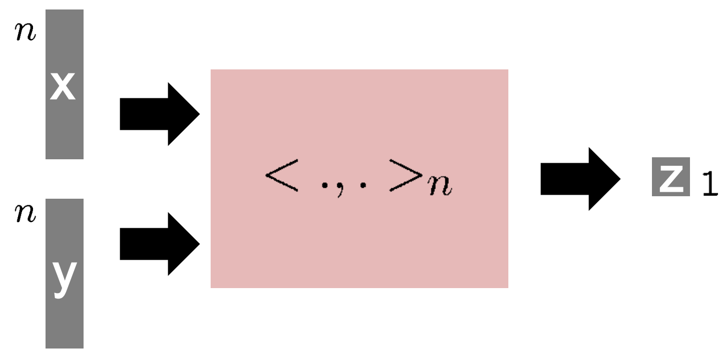
- Operator: Multiple vectors ! Multiple vectors
- Stateless
- Higher-dimensional data is linearized
- Operators are potentially nonlinear

$$\mathrm{M}: \begin{cases} \mathbb{C}^{n_0} \times \cdots \times \mathbb{C}^{n_{k-1}} \to \mathbb{C}^{N_0} \times \cdots \times \mathbb{C}^{N_{\ell-1}} \\ (\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}) \mapsto \mathrm{M}(\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}) \end{cases}$$

Example: Scalar product

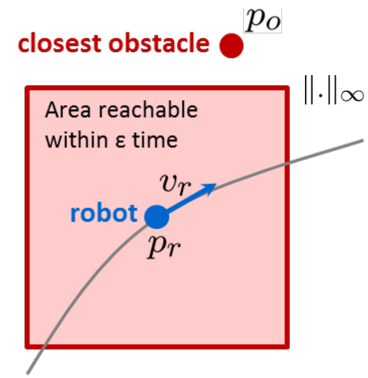
$$< .,. >_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$

$$\big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto \sum_{i=0}^{n-1} x_i y_i$$



F. Franchetti, F. de Mesmay, Daniel McFarlin, and M. Püschel, **"Operator Language: A Program Generation Framework for Fast Kernels,"** Proceedings of IFIP Working Conference on Domain Specific Languages (DSL WC), 2009. ***Best Paper Award.***

Example: Safety Distance as OL Operator



- **Passive Safety of Robots**
  p_o: Position of closest obstacle
  p_r: Position of robot
  v_r: Longitudinal velocity of robot
  A , b, V, " : constants

$$\|p_r - p_o\|_\infty > \frac{v_r^2}{2b} + V\frac{v_r}{b} + \left(\frac{A}{b} + 1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon(v_r + V)\right)$$

- **Definition as operator**

$$\mathsf{SafeDist}_{V,A,b,\varepsilon} : \quad \mathbb{R} \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{Z}_2$$
$$(v_r, p_r, p_o) \mapsto \big(p(v_r) < d_\infty(p_r, p_o)\big) \quad \text{with} \quad d_\infty(\vec{x}, \vec{y}) = \|\vec{x} - \vec{y}\|_\infty$$

$$p(x) = \alpha x^2 + \beta x + \gamma$$
$$\alpha = \frac{1}{2b}$$
$$\beta = \frac{V}{b} + \varepsilon\left(\frac{A}{b} + 1\right)$$
$$\gamma = \left(\frac{A}{b} + 1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon V\right)$$

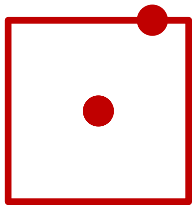
Formalizing Mathematical Objects in OL

- **Infinity norm**

$$\|.\|_\infty^n : \mathbb{R}^n \to \mathbb{R}$$
$$(x_i)_{i=0,\ldots,n-1} \mapsto \max_{i=0,\ldots,n-1} |x_i|$$

- **Chebyshev distance**

$$d_\infty^n(.,.) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$(x, y) \mapsto \|x - y\|_\infty^n$$



- **Vector subtraction**

$$(-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$(x, y) \mapsto x - y$$

- **Pointwise comparison**

$$(<)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{Z}_2^n$$
$$\big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto (x_i < y_i)_{i=0,\ldots,n-1}$$

- **Scalar product**

$$< .,. >_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$\big((x_i)_{i=0,\ldots,n-1}, (y_i)_{i=0,\ldots,n-1}\big) \mapsto \sum_{i=0}^{n-1} x_i y_i$$

- **Monomial enumerator**

$$(x^i)_n : \mathbb{R} \to \mathbb{R}^{n+1}$$
$$x \mapsto (x^i)_{i=0,\ldots,n}$$

- **Polynomial evaluation**

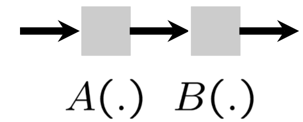
$$P[x, (a_0, \ldots, a_n)] : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n$$

Beyond the textbook: explicit vector length, infix operators as prefix operators

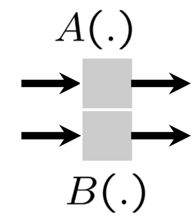
Operations and Operator Expressions

- **Operations (higher-order operators)**

$$\circ : (D \to S) \times (S \to R) \to (D \to R)$$
$$(A, B) \mapsto B \circ A$$

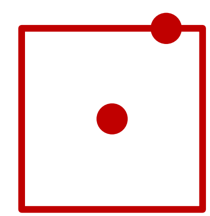


$$\times : (D \to R) \times (E \to S) \to (D \times E \to R \times S)$$
$$(A, B) \mapsto \big((x, y) \mapsto \big(A(x), B(y)\big)\big)$$



- **Operator expressions are operators**

$$\|.\|_\infty^n \circ (-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$
$$\big((x_i)_{i=0,\dots,n-1}, (y_i)_{i=0,\dots,n-1}\big) \mapsto \max_{i=0,\dots,n-1} |x_i - y_i|$$

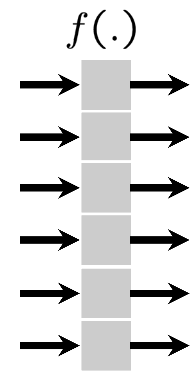


- **Short-hand notation: Infix notation**

$$A(.) - B(.) = \big(x \mapsto A(x) - B(x)\big)$$ can be expressed via $$(-)_n : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$$
$$(x, y) \mapsto x - y$$

Basic OL Operators

- **Basic operators ≈ functional programming constructs**



map	$\mathsf{Pointwise}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}^n$ $(x_i)_i \mapsto f_0(x_0) \oplus \cdots \oplus f_{n-1}(x_{n-1})$
binop	$\mathsf{Atomic}_{f(.,.)} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ $(x, y) \mapsto f(x, y)$
map + zip	$\mathsf{Pointwise}_{n\times n,f_i} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ $\big((x_i)_i, (y_i)_i\big) \mapsto f_0(x_0, y_0) \oplus \cdots \oplus f_{n-1}(x_{n-1}, y_{n-1})$
fold	$\mathsf{Reduction}_{n,f_i} : \mathbb{R}^n \to \mathbb{R}$ $(x_i)_i \mapsto f_{n-1}(x_{n-1}, f_{n-2}(x_{n-2}, f_{n-3}(\ldots f_0(x_0, \mathsf{id}()) \ldots)$
unfold	$\mathsf{Induction}_{n,f_i} : \mathbb{R} \to \mathbb{R}^{n+1}$ $x \mapsto (f_n(x, f_{n-1}(\ldots) \ldots), \ldots, f_2(x, f_1(x, \mathsf{id})), f_1(x, \mathsf{id}), \mathsf{id}())$

- **Safety distance as (optimized) operator expression**

$$\mathsf{SafeDist}_{V,A,b,\varepsilon} = \mathsf{Atomic}_{(x,y)\mapsto x<y} \circ \Big(\Big(\mathsf{Reduction}_{3,(x,y)\mapsto x+y} \circ \mathsf{Pointwise}_{3,x\mapsto a_i x} \circ \mathsf{Induction}_{3,(a,b)\mapsto ab,1}\Big) \times \Big(\mathsf{Reduction}_{2,(x,y)\mapsto \max(|x|,|y|)} \circ \mathsf{Pointwise}_{2\times 2,(x,y)\mapsto x-y}\Big)\Big)$$

Breaking Down Operators into Expressions

- **Application specific:** Safety Distance as Rewrite Rule

$$\mathsf{SafeDist}_{V,A,b,\varepsilon}(.,.,.) \to \Big(P[x,(a_0,a_1,a_2)](.) < d_\infty^2(.,.)\Big)(.,.,.)$$

with $a_0 = \frac{1}{2b},\ a_1 = \frac{V}{b} + \varepsilon\left(\frac{A}{b}+1\right),\ a_2 = \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon V\right)$

Problem specification: hand-developed or automatically produced

- **One-time effort:** mathematical library

$$d_\infty^n(.,.) \to ||.||_\infty^n \circ (-)_n$$

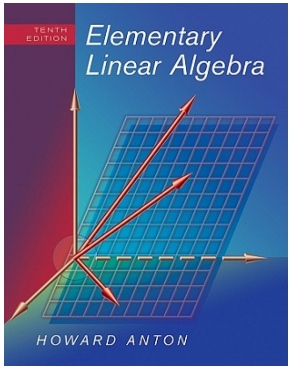
$$(\diamond)_n \to \mathsf{Pointwise}_{n\times n,(a,b)\mapsto a\diamond b}, \quad \diamond \in \{+,-\cdot,\wedge,\vee,\dots\}$$

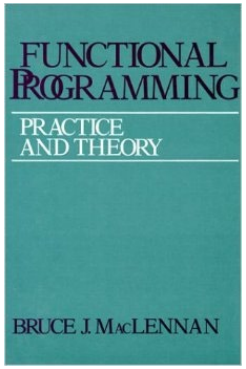
$$||.||_\infty^n \to \mathsf{Reduction}_{n,(a,b)\mapsto \mathsf{max}(|a|,|b|)}$$

$$< .,. >_n \to \mathsf{Reduction}_{n,(a,b)\mapsto a+b} \circ \mathsf{Pointwise}_{n\times n,(a,b)\mapsto ab}$$

$$P[x,(a_0,\dots,a_n)] \to < (a_0,\dots,a_n),. > \circ (x^i)_n$$

$$(x^i)_n \to \mathsf{Induction}_{n,(a,b)\mapsto ab,1}$$





Library of well-known identities expressed in OL

Inspiration: Symbolic Integration

- **Rule based AI system**
  basic functions, substitution
- **May not succeed**
  not all expressions can be symbolically integrated
- **Arbitrarily extensible**
  define new functions as integrals
  Γ(.), distributions, Lebesgue integral
- **Semantics preserving**
  rule chain = formal proof
- **Automation**
  Mathematica, Maple

Table of Integrals

BASIC FORMS

(1) $\int x^n dx = \frac{1}{n+1} x^{n+1}$

(2) $\int \frac{1}{x} dx = \ln x$

(3) $\int u dv = uv - \int v du$

(4) $\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx$

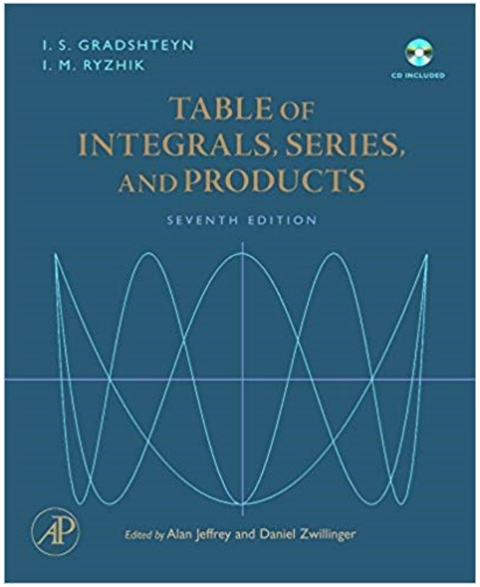
RATIONAL FUNCTIONS

(5) $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax+b)$

(6) $\int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a}$

(7) $\int (x+a)^n dx = (x+a)^n \left(\frac{a}{1+n} + \frac{x}{1+n} \right), \ n \neq -1$

(8) $\int x(x+a)^n dx = \frac{(x+a)^{1+n}(nx+x-a)}{(n+2)(n+1)}$



In[31]:= $\int_0^{2\pi} \frac{1}{a^2 \text{Cos}[t]^2 + b^2 \text{Sin}[t]^2} dt$

Out[31]= $\frac{2 \sqrt{\frac{b^2}{a^2}} \pi}{b^2}$

In[33]:= $\int_0^{2\pi} \frac{1}{a^2 \left(\frac{e^{i t} + e^{-i t}}{2}\right)^2 + b^2 \left(\frac{e^{i t} - e^{-i t}}{2 i}\right)^2} dt$

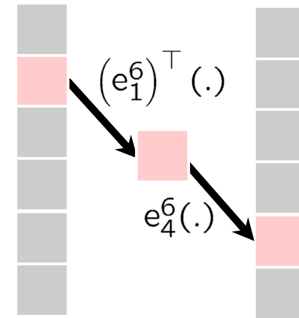
Out[33]= 0

Σ–OL: Low-Level Operator Language

- **Selection and embedding operator:** ***gather and scatter***

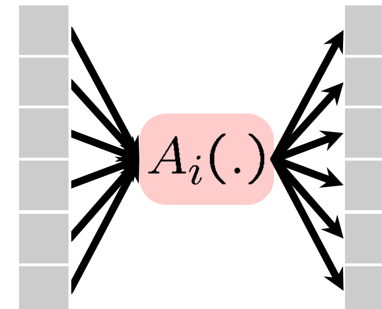
$$(\mathsf{e}_i^n)^\top(.) : \mathbb{R}^n \to \mathbb{R}^1$$
$$(x_i)_{i=0,\ldots,n-1} \mapsto x_i$$

$$\mathsf{e}_i^n(.) : \mathbb{R}^1 \to \mathbb{R}^n$$
$$(x) \mapsto (0,\ldots,0,\underbrace{x}_{i\text{th}},0,\ldots,0)$$



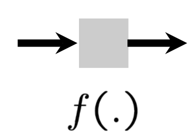
- **Iterative operations:** ***loop***

$$\bigsqcup_{i=0}^{n-1} : (D \to R)^n \to (D \to R)$$
$$A_i \mapsto (x \mapsto A_0(x) \sqcup \cdots \sqcup A_{n-1}(x))$$
$$\text{with } \sqcup \in \left\{\sum, \vee, \wedge, \prod, \min, \max, \ldots\right\}$$



- **Atomic operators:** ***nonlinear scalar functions***

$$\text{Atomic}_f : \mathbb{R}^1 \to \mathbb{R}^1$$
$$(x) \mapsto (f(x))$$



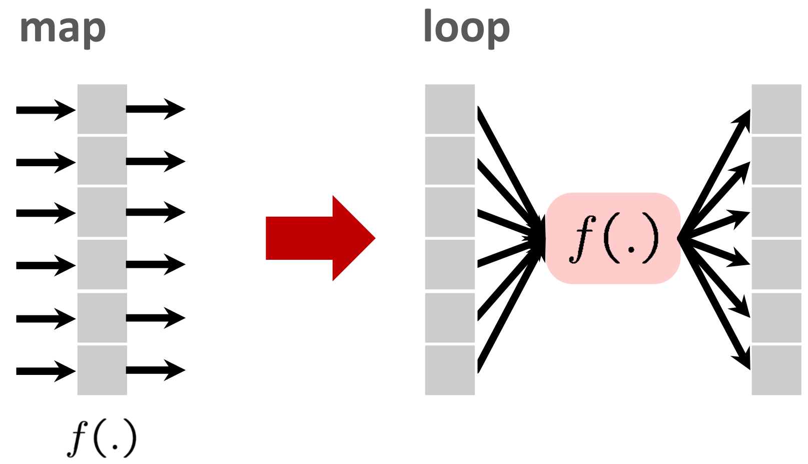
Σ-OL operator expressions = array-based programs with for loops

Rule-Based Translation and Optimization

- **Translating Basic OL into Σ-OL**

$$\text{Pointwise}_{n,f_i} \to \sum_{i=0}^{n-1} \left(\text{e}_i^n \circ \text{Atomic}_{f_i} \circ (\text{e}_i^n)^\top\right)$$

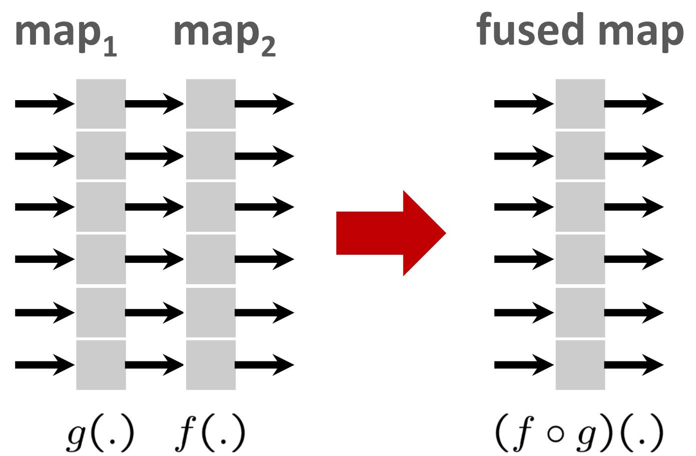
$$\text{Reduction}_{n,(a,b)\mapsto a+b} \to \sum_{i=0}^{n-1} (\text{e}_i^n)^\top$$



- **Optimizing Basic OL/Σ-OL**

$$\text{Pointwise}_{n,f_i} \circ \text{Pointwise}_{n,g_i} \to \text{Pointwise}_{n,f_i \circ g_i}$$

$$\text{Pointwise}_{n,f_i} \circ \text{e}_n^j \to \text{e}_n^j \circ \text{Pointwise}_{1,f_j}$$



Captures program optimizations that are traditionally hard to do

Last Step: Abstract Code

Code objects

- Values and types
- Arithmetic operations
- Logic operations
- Constants, arrays and scalar variables
- Assignments and control flow

Properties: at the same time

- Program = (abstract syntax) tree
- Represents program in restricted C
- OL operator over real numbers and machine numbers (floating-point)
- Pure functional interpretation
- Represents lambda expression

```
# Dynamic Window Monitor

let(
  i3 := var("i3", TInt), i5 := var("i5", TInt),
  w2 := var("w2", TBool), w1 := var("w1", T_Real(64)),
  s8 := var("s8", T_Real(64)), s7 := var("s7", T_Real(64)),
  s6 := var("s6", T_Real(64)), s5 := var("s5", T_Real(64)),
  s4 := var("s4", T_Real(64)), s1 := var("s1", T_Real(64)),
  q4 := var("q4", T_Real(64)), q3 := var("q3", T_Real(64)),
  D := var("D", TPtr(T_Real(64)).aligned([16, 0])),
  X := var("X", TPtr(T_Real(64)).aligned([16, 0])),

  func(TInt, "dwmonitor", [ X, D ],
     decl([q3, q4, s1, s4, s5, s6, s7, s8, w1, w2],
        chain(
           assign(s5, V(0.0)),
           assign(s8, nth(X, V(0))),
           assign(s7, V(1.0)),
           loop(i5, [0..2],
              chain(
                 assign(s4, mul(s7, nth(D, i5))),
                 assign(s5, add(s5, s4)),
                 assign(s7, mul(s7, s8))
              )
           ),
           assign(s1, V(0.0)),
           loop(i3, [0..1],
              chain(
                 assign(q3, nth(X, add(i3, V(1)))),
                 assign(q4, nth(X, add(V(3), i3))),
                 assign(w1, sub(q3, q4)),
                 assign(s6, cond(geq(w1, V(0)), w1, neg(w1))),
                 assign(s1, cond(geq(s1, s6), s1, s6))
              )
           ),
           assign(w2, geq(s1, s5)),
           creturn(w2)
        )
     )
  )
)
```

Translating Σ–OL to Abstract Code

Compilation rules: recursive descent

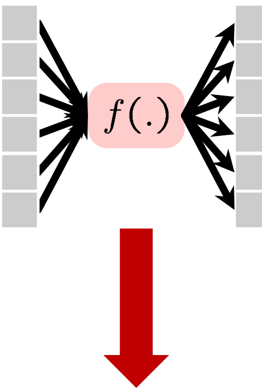
$$\mathsf{Code}\left(y = (A \circ B)(x)\right) \;\rightarrow\; \left\{\mathtt{decl}(t), \mathsf{Code}\left(t = B(x)\right), \mathsf{Code}\left(y = A(t)\right)\right\}$$

$$\mathsf{Code}\left(y = \left(\sum_{i=0}^{n-1} A_i\right)(x)\right) \;\rightarrow\; \left\{y := \vec{0}, \mathtt{for}(i = 0..n-1)\ \mathsf{Code}\left(y += A_i(x)\right)\right\}$$

$$\mathsf{Code}\left(y = (\mathrm{e}_i^n)^\top(x)\right) \;\rightarrow\; y[0] := x[i]$$

$$\mathsf{Code}\left(y = \mathrm{e}_i^n(x)\right) \;\rightarrow\; \left\{y = \vec{0}, y[i] := x[0]\right\}$$

$$\mathsf{Code}\left(y = \mathsf{Atomic}_f(x)\right) \;\rightarrow\; y[0] := f(x[i])$$



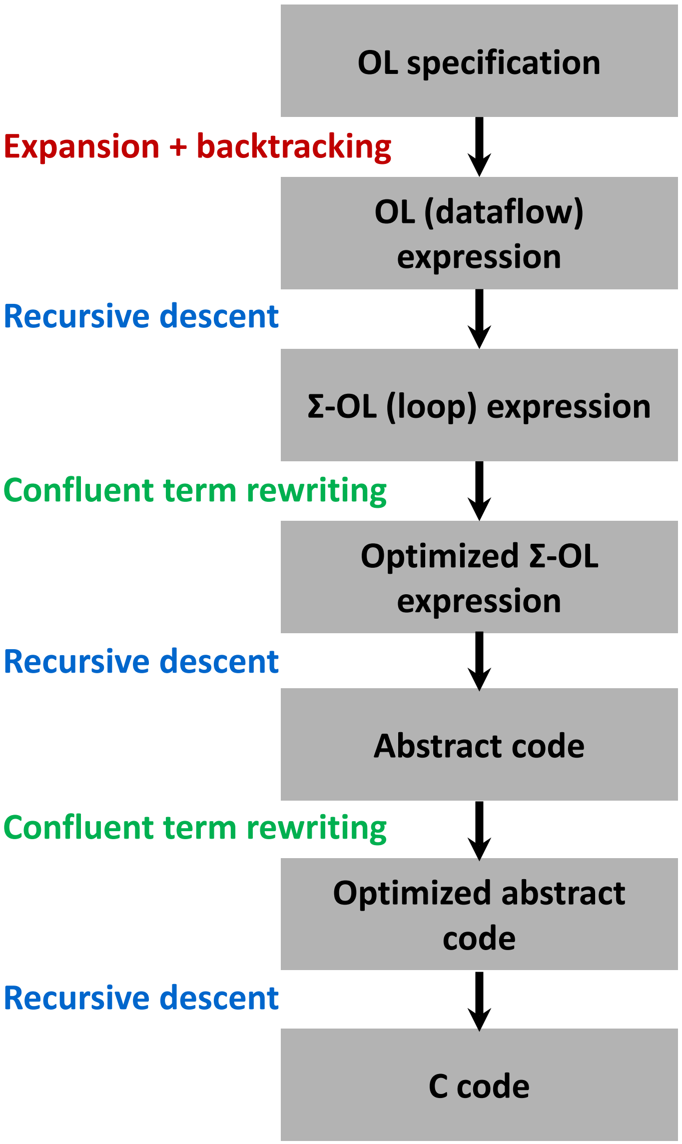

```
chain(
  assign(Y, V(0.0),
  loop(i1, [0..5],
    assign(nth(y, i1),
      f(nth(X, i1)))
  )
)
```

Cleanup rules: term rewriting

```
chain(a, chain(b))    → chain([a, b])
decl(D, decl(E, c))   → decl([D, E], c)
loop(i, decl(D, c))   → decl(D, loop(i, c))
chain(a, decl(D, b))  → decl(D, chain([a, b]))
```

Rule-based code generation and backend compilation

Putting it Together: One Big Rule System



Mathematical specification

$$\text{SafeDist}_{V,A,b,\varepsilon}(.,.,.) \to \left(P[x,(a_0,a_1,a_2)](.) < d_\infty^2(.,.)\right)(.,.,.)$$

with $a_0=\frac{1}{2b},\ a_1=\frac{V}{b}+\varepsilon\left(\frac{A}{b}+1\right),\ a_2=\left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon V\right)$

Final code

```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8 , x1, x10, x13, x14, x17
    int w1;
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN + DBL_MIN)), _mm_lo
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
        x3 = _mm_mul_pd(_mm_shuffle_pd(x1, x1, _MM_SHUFFLE2(0, 1)),
        x4 = _mm_sub_pd(_mm_set1_pd(0.0), _mm_min_pd(x3, x2));
        u3 = _mm_add_pd(_mm_max_pd(_mm_shuffle_pd(x4, x4, _MM_SHUFF
```

Final Synthesized C Code

```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8 , x1, x10, x13, x14, x17, x18, x19, x2, x3, x4, x6, x7, x8, x9;
    int w1;
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[0])));
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN + DBL_MIN)), _mm_loaddup_pd(&(D[i5])));
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
        x3 = _mm_mul_pd(_mm_shuffle_pd(x1, x1, _MM_SHUFFLE2(0, 1)), x6);
```

$$\begin{aligned}\mathsf{SafeDist}_{V,A,b,\varepsilon} = &\ \mathsf{Atomic}_{(x,y)\mapsto x<y} \\ &\circ \Big(\Big(\mathsf{Reduction}_{3,(x,y)\mapsto x+y} \circ \mathsf{Pointwise}_{3,x\mapsto a_i x} \circ \mathsf{Induction}_{3,(a,b)\mapsto ab,1} \Big) \\ &\quad \times \Big(\mathsf{Reduction}_{2,(x,y)\mapsto \max(|x|,|y|)} \circ \mathsf{Pointwise}_{2\times 2,(x,y)\mapsto x-y} \Big) \Big)\end{aligned}$$

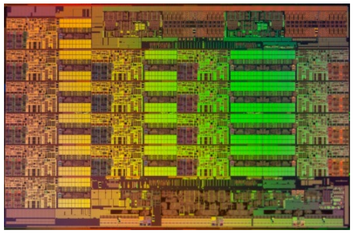
```
    }
    u6
    for
        u8 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(i3 + 1)])));
        u7 = _mm_cvtps_pd(_mm_addsub_ps(_mm_set1_ps(FLT_MIN), _mm_set1_ps(X[(3 + i3)])));
        x14 = _mm_add_pd(u8, _mm_shuffle_pd(u7, u7, _MM_SHUFFLE2(0, 1)));
        x13 = _mm_shuffle_pd(x14, x14, _MM_SHUFFLE2(0, 1));
        u4 = _mm_shuffle_pd(_mm_min_pd(x14, x13), _mm_max_pd(x14, x13), _MM_SHUFFLE2(1, 0));
        u6 = _mm_shuffle_pd(_mm_min_pd(u6, u4), _mm_max_pd(u6, u4), _MM_SHUFFLE2(1, 0));
    }
    x17 = _mm_addsub_pd(_mm_set1_pd(0.0), u6);
    x18 = _mm_addsub_pd(_mm_set1_pd(0.0), u5);
    x19 = _mm_cmpge_pd(x17, _mm_shuffle_pd(x18, x18, _MM_SHUFFLE2(0, 1)));
    w1 = (_mm_testc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffffff)) –
         (_mm_testnzc_si128(_mm_castpd_si128(x19), _mm_set_epi32(0xffffffff, 0xffffffff, 0xffffffff, 0xffffffff))));
    __asm nop;
    if (_mm_getcsr() & 0x0d) {
        _mm_setcsr(_xm);
        return -1;
    }
    _mm_setcsr(_xm);
    return w1;
}
```

Outline

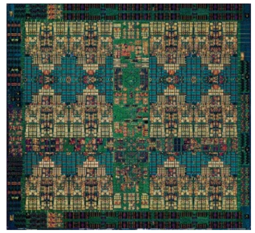
- Introduction
- Operator Language
- **Achieving Performance Portability**
- FFTX: A Library Frontend for SPIRAL
- Summary

Today's Computing Landscape

1 Gflop/s = one billion floating-point operations (additions or multiplications) per second

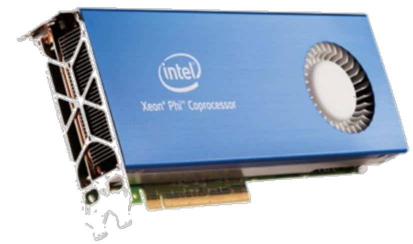


Intel Xeon 8180M
2.25 Tflop/s, 205 W
28 cores, 2.5—3.8 GHz
2-way—16-way AVX-512



IBM POWER9
768 Gflop/s, 300 W
24 cores, 4 GHz
4-way VSX-3

Nvidia Tesla V100
7.8 Tflop/s, 300 W
5120 cores, 1.2 GHz
32-way SIMT



Intel Xeon Phi 7290F
1.7 Tflop/s, 260 W
72 cores, 1.5 GHz
8-way/16-way LRBni

Snapdragon 835
15 Gflop/s, 2 W
8 cores, 2.3 GHz
A540 GPU, 682 DSP, NEON

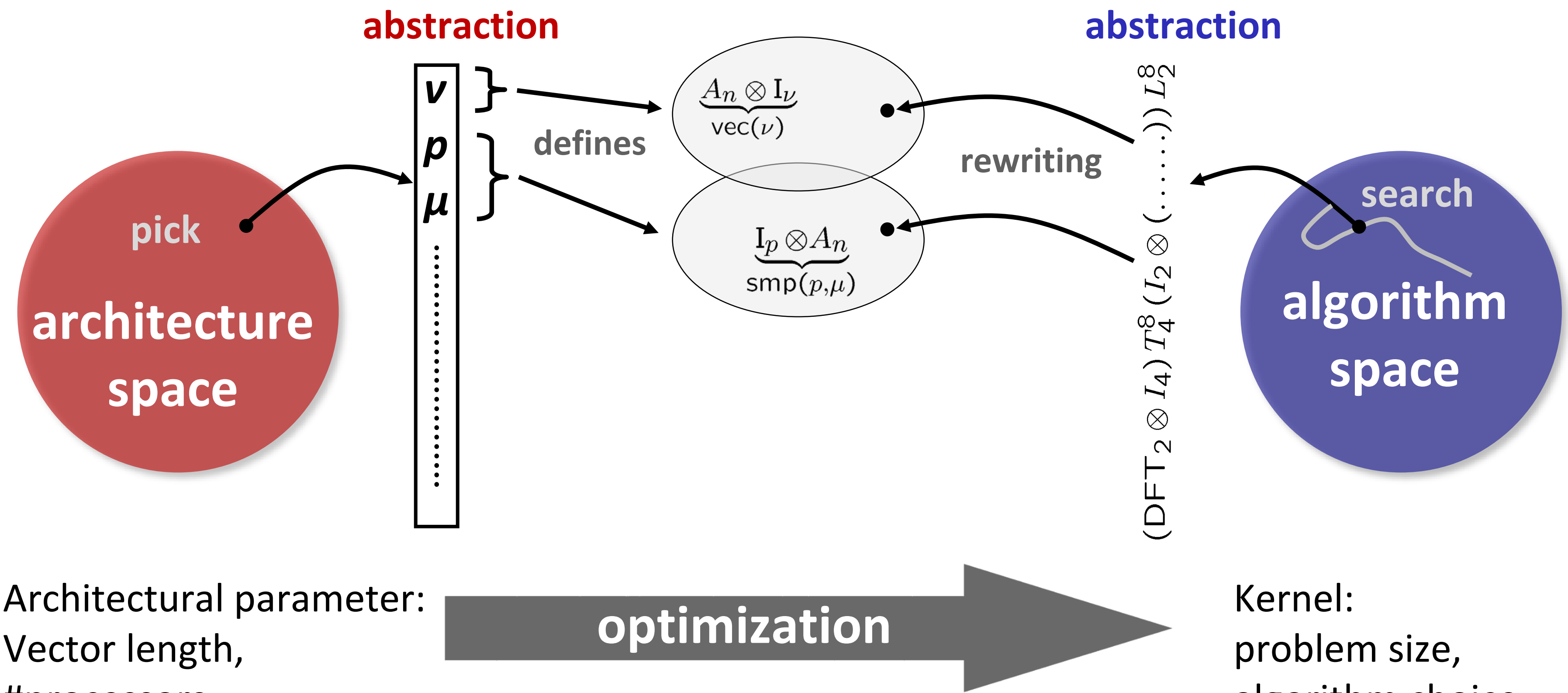
Intel Atom C3858
32 Gflop/s, 25 W
16 cores, 2.0 GHz
2-way/4-way SSSE3

Dell PowerEdge R940
3.2 Tflop/s, 6 TB, 850 W
4x 24 cores, 2.1 GHz
4-way/8-way AVX

Summit
187.7 Pflop/s, 13 MW
9,216 x 22 cores POWER9
+ 27,648 V100 GPUs

Platform-Aware Formal Program Synthesis

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

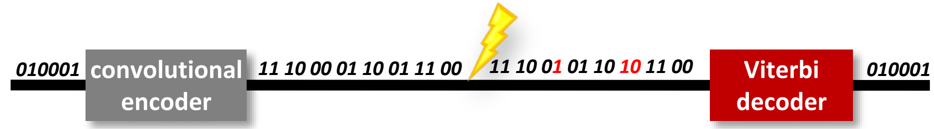
Kernel:
problem size,
algorithm choice

Some Application Domains in OL

Linear Transforms

$$\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n (\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \gcd(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\, \mathrm{L}^n_m (\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\quad \cdot (\mathrm{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m) \left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m \right) \right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathrm{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathrm{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathrm{J}_2\, \mathrm{R}_{13\pi/8}
\end{aligned}$$

Software Defined Radio



$$F_{K,F} \rightarrow \prod_{i=1}^{F} \left((I_{2^{K-2}} \otimes_j B_{F-i,j}) L^{2^{K-1}}_{2^{K-2}} \right)$$

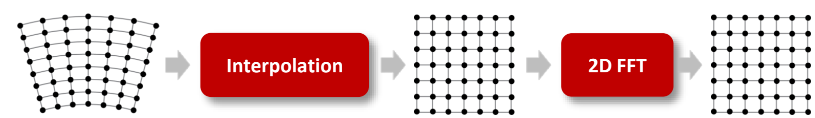
$$\underline{\mathbf{F}_{K,F}}\ \nu \rightarrow \prod_{i=1}^{F} \left(\left(\mathrm{I}_{2^{K-2}/\nu} \otimes_{j_1} \vec{\mathrm{L}}^{2\nu}_{\nu} \vec{B}^{\nu}_{F-i,j_1} \right) \left(\mathrm{L}^{2^{K-1}/\nu}_{2^{K-2}/\nu} \bar{\otimes} \mathrm{I}_\nu \right) \right)$$

$$B_{i,j} : \begin{cases} \pi_U = \min_{d_U}(\pi_A + \beta_{A \to U}, \pi_B + \beta_{B \to U}) \\ \pi_V = \min_{d_V}(\pi_A + \beta_{A \to V}, \pi_B + \beta_{B \to V}) \end{cases}$$

Matrix-Matrix Multiplication

$$\begin{aligned}
&\mathrm{MMM}_{1,1,1} \rightarrow (\cdot)_1 \\
&\mathrm{MMM}_{m,n,k} \rightarrow (\otimes)_{m/m_b \times 1} \otimes \mathrm{MMM}_{m_b,n,k} \\
&\mathrm{MMM}_{m,n,k} \rightarrow \mathrm{MMM}_{m,nb,k} \otimes (\otimes)_{1 \times n/nb} \\
&\mathrm{MMM}_{m,n,k} \rightarrow ((\Sigma_{k/k_b} \circ (\cdot)_{k/k_b}) \otimes \mathrm{MMM}_{m,n,k_b}) \circ \\
&\qquad ((L^{mk/k_b}_{k/k_b} \otimes I_{k_b}) \times I_{kn}) \\
&\mathrm{MMM}_{m,n,k} \rightarrow (L^{mn/n_b}_{m} \otimes I_{n_b}) \circ \\
&\qquad ((\otimes)_{1 \times n/n_b} \otimes \mathrm{MMM}_{m,n_b,k}) \circ \\
&\qquad (I_{km} \times (L^{kn/n_b}_{n/n_b} \otimes I_{n_b}))
\end{aligned}$$

Synthetic Aperture Radar (SAR)



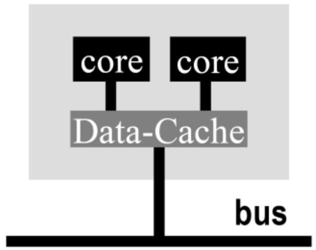
$$\begin{aligned}
\mathrm{SAR}_{k \times m \to n \times n} &\rightarrow \mathrm{DFT}_{n \times n} \circ \mathrm{Interp}_{k \times m \to n \times n} \\
\mathrm{DFT}_{n \times n} &\rightarrow (\mathrm{DFT}_n \otimes \mathrm{I}_n) \circ (\mathrm{I}_n \otimes \mathrm{DFT}_n) \\
\mathrm{Interp}_{k \times m \to n \times n} &\rightarrow (\mathrm{Interp}_{k \to n} \otimes_i \mathrm{I}_n) \circ (\mathrm{I}_k \otimes_i \mathrm{Interp}_{m \to n}) \\
\mathrm{Interp}_{r \to s} &\rightarrow \left(\bigoplus_{i=0}^{n-2} \mathrm{InterpSeg}_k \right) \oplus \mathrm{InterpSegPruned}_{k,\ell} \\
\mathrm{InterpSeg}_k &\rightarrow \mathrm{G}_f^{u \cdot n \to k} \circ \mathrm{iPrunedDFT}_{n \to u \cdot n} \circ \left(\frac{1}{n} \right) \circ \mathrm{DFT}_n
\end{aligned}$$

Formal Approach for all Types of Parallelism

- **Multithreading** (Multicore) $\mathrm{I}_p \otimes_{\|} A_{\mu n}, \quad \mathbf{L}_m^{mn} \bar{\otimes} \mathrm{I}_\mu$
- **Vector SIMD** (SSE, VMX/Altivec,...) $A \hat{\otimes} \mathrm{I}_\nu \quad \underbrace{\mathbf{L}_2^{2\nu}}_{\text{isa}}, \quad \underbrace{\mathbf{L}_\nu^{2\nu}}_{\text{isa}}, \quad \underbrace{\mathbf{L}_\nu^{\nu^2}}_{\text{isa}}$
- **Message Passing** (Clusters, MPP) $\mathrm{I}_p \otimes_{\|} A_n, \quad \underbrace{\mathbf{L}_p^{p^2} \bar{\otimes} \mathrm{I}_{n/p^2}}_{\text{all-to-all}}$
- **Streaming/multibuffering** (Cell) $\mathrm{I}_n \otimes_2 A_{\mu n}, \quad \mathbf{L}_m^{mn} \bar{\otimes} \mathrm{I}_\mu$
- **Graphics Processors** (GPUs) $\prod_{i=0}^{n-1} A_i, \quad A_n \hat{\otimes} \mathrm{I}_w, \quad P_n \otimes Q_w$
- **Gate-level parallelism** (FPGA) $\prod_{i=0}^{n-1}{}^{\text{ir}} A, \quad \mathrm{I}_s \tilde{\otimes} A, \quad \underbrace{\mathbf{L}_n^m}_{\text{bram}}$
- **HW/SW partitioning** (CPU + FPGA) $\underbrace{A_1}_{\text{fpga}}, \quad \underbrace{A_2}_{\text{fpga}}, \quad \underbrace{A_3}_{\text{fpga}}, \quad \underbrace{A_4}_{\text{fpga}}$

Modeling Hardware: Base Cases

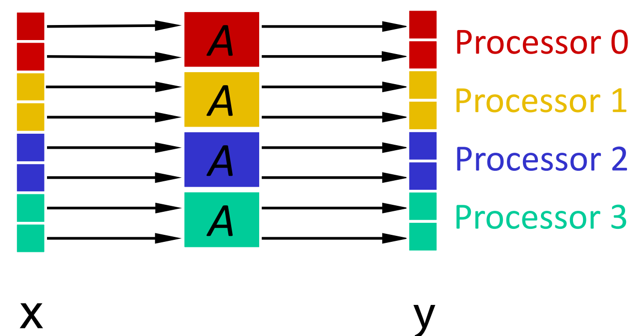
- **Hardware abstraction: shared cache with cache lines**



$$\underbrace{A_{k\times m\rightarrow n}}_{\mathsf{smp}(p,\mu)}$$

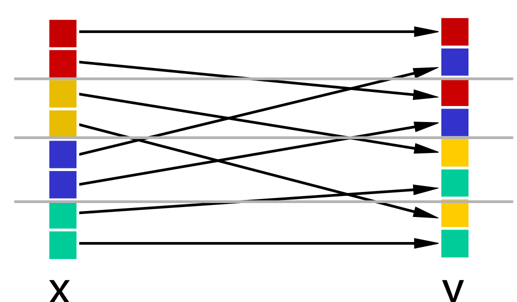
- **Tensor product: embarrassingly parallel operator**

$$y = \left(\mathrm{I}_p \otimes A\right)(x)$$



- **Permutation: problematic; may produce false sharing**

$$y = \mathrm{L}^8_4(x)$$



Example Program Transformation Rule Set

$$\underbrace{AB}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{A}_{\mathsf{smp}(p,\mu)} \underbrace{B}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{A_m \otimes \mathrm{I}_n}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{\left(\mathsf{L}_m^{mp} \otimes \mathrm{I}_{n/p}\right)\left(\mathrm{I}_p \otimes (A_m \otimes \mathrm{I}_{n/p})\right)\left(\mathsf{L}_p^{mp} \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{\mathsf{L}_m^{mn}}_{\mathsf{smp}(p,\mu)} \rightarrow \begin{cases} \underbrace{\left(\mathrm{I}_p \otimes \mathsf{L}_{m/p}^{mn/p}\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathsf{L}_p^{pn} \otimes \mathrm{I}_{m/p}\right)}_{\mathsf{smp}(p,\mu)} \\ \underbrace{\left(\mathsf{L}_m^{pm} \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathrm{I}_p \otimes \mathsf{L}_m^{mn/p}\right)}_{\mathsf{smp}(p,\mu)} \end{cases}$$

Recursive rules

$$\underbrace{\mathrm{I}_m \otimes A_n}_{\mathsf{smp}(p,\mu)} \rightarrow \mathrm{I}_p \otimes_{\parallel} \left(\mathrm{I}_{m/p} \otimes A_n\right)$$

$$\underbrace{(P \otimes \mathrm{I}_n)}_{\mathsf{smp}(p,\mu)} \rightarrow \left(P \otimes \mathrm{I}_{n/\mu}\right) \overline{\otimes}\, \mathrm{I}_\mu$$

Base case rules

Autotuning in Constraint Solution Space

$\underbrace{\mathrm{DFT}_8}_{\mathrm{AVX(2\text{-}way}\ \mathbb{C})}$

AVX 2-way `_Complex double`

DFT_8

Base cases

$A^{n\times n} \vec{\otimes} \mathrm{I}_2$

$\underbrace{\mathrm{L}_2^4}_{\mathrm{vec}(2)}$

$\underbrace{\mathrm{T}_n^{mn}}_{\mathrm{vec}(2)}$

Transformation rules

$$(\mathrm{I}_m \otimes A^{n\times n})\mathrm{L}_m^{mn} \to \big(\mathrm{I}_{m/\nu} \otimes \mathrm{L}_\nu^{n\nu}(A^{n\times n} \otimes \mathrm{I}_\nu)\big)(\mathrm{L}_{m/\nu}^{mn/\nu} \otimes \mathrm{I}_\nu)$$

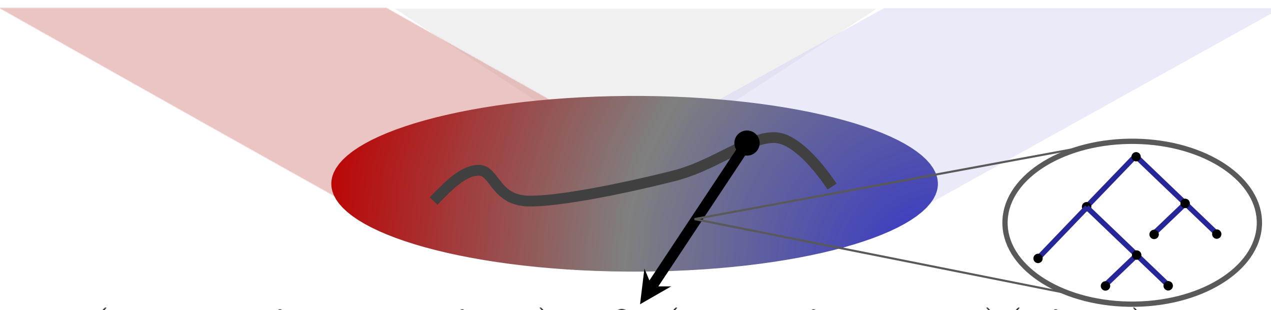
$$\mathrm{L}_\nu^{n\nu} \to (\mathrm{L}_\nu^n \otimes \mathrm{I}_\nu)(\mathrm{I}_{n/\nu} \otimes \mathrm{L}_\nu^{\nu^2})$$

$$A^{m\times m} \otimes \mathrm{I}_n \to (A^{m\times m} \otimes \mathrm{I}_{n/\nu}) \otimes \mathrm{I}_\nu$$

Breakdown rules

$$\mathrm{DFT}_{mn} \to (\mathrm{DFT}_m \otimes \mathrm{I}_n)\mathrm{T}_n^{mn}(\mathrm{I}_m \otimes \mathrm{DFT}_n)\mathrm{L}_m^{mn}$$

$$\mathrm{DFT}_2 \to \mathrm{F}_2$$



$$\Big((\mathrm{F}_2 \otimes \mathrm{I}_2)\mathrm{T}_2^4(\mathrm{I}_2 \otimes \mathrm{F}_2)\mathrm{L}_2^4 \vec{\otimes} \mathrm{I}_2\Big) \underbrace{\mathrm{T}_2^8}_{\mathrm{vec}(2)} \Big(\mathrm{I}_2 \otimes \underbrace{\mathrm{L}_2^4}_{\mathrm{vec}(2)} (\mathrm{F}_2 \vec{\otimes} \mathrm{I}_2)\Big)\Big(\mathrm{L}_2^4 \vec{\otimes} \mathrm{I}_2\Big)$$

OL specification

↓ Expansion + backtracking

OL (dataflow) expression

↓ Recursive descent

Σ-OL (loop) expression

↓ Confluent term rewriting

Optimized Σ-OL expression

↓ Recursive descent

Abstract code

↓ Confluent term rewriting

Optimized abstract code

↓ Recursive descent

C code

Translating an OL Expression Into Code

Constraint Solver Input: $\underbrace{\mathrm{DFT}_8}_{\mathrm{AVX}(2\text{-way }\mathbb{C})}$

Output =

Ruletree, expanded into

OL Expression:

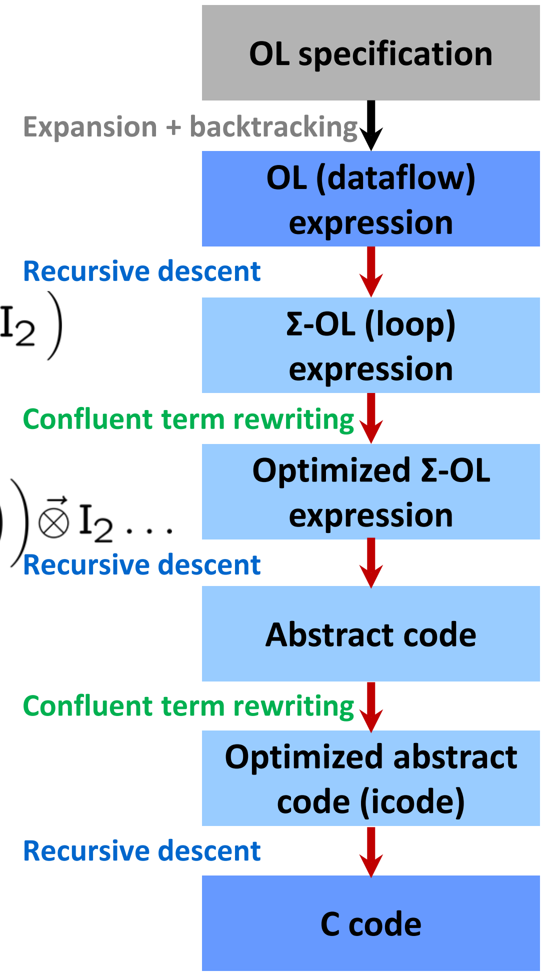
$$\left((F_2 \otimes I_2) T^4_2 (I_2 \otimes F_2) L^4_2 \vec{\otimes} I_2\right) \underbrace{T^8_2}_{\mathrm{vec}(2)} \left(I_2 \otimes \underbrace{L^4_2}_{\mathrm{vec}(2)} (F_2 \vec{\otimes} I_2)\right)\left(L^4_2 \vec{\otimes} I_2\right)$$

Σ-OL:

$$\left(\sum_{j=0}^{1}\left(S_{\imath_2\otimes(j)_2} F_2 \mathrm{Map}^2_{x\mapsto\omega_4^{2i+j}x} G_{\imath_2\otimes(j)_2}\right)\sum_{j=0}^{1}\left(S_{(j)_2\otimes\imath_2} F_2 G_{\imath_2\otimes(j)_2}\right)\right)\vec{\otimes} I_2 \ldots$$

C Code:

```
void dft8(_Complex double *Y, _Complex double *X) {
     __m256d s38, s39, s40, s41,...
     __m256d  *a17, *a18;
     a17 = ((__m256d  *) X);
     s38 = *(a17);
     s39 = *((a17 + 2));
     t38 = _mm256_add_pd(s38, s39);
     t39 = _mm256_sub_pd(s38, s39);
     ...
     s52 = _mm256_sub_pd(s45, s50);
     *((a18 + 3)) = s52;
}
```



Symbolic Verification for Linear Operators

- **Linear operator = matrix-vector product**
  Algorithm = matrix factorization

$$\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}=\begin{bmatrix}1&\cdot&1&\cdot\\\cdot&1&\cdot&1\\1&\cdot&-1&\cdot\\\cdot&1&\cdot&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&\cdot&\cdot&j\end{bmatrix}\begin{bmatrix}1&1&\cdot&\cdot\\1&-1&\cdot&\cdot\\\cdot&\cdot&1&1\\\cdot&\cdot&1&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&\cdot&1\end{bmatrix}\quad = ?$$

$$\mathrm{DFT}_4 = (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\,\mathrm{T}^4_2\,(\mathrm{I}_2 \otimes \mathrm{DFT}_2)\,\mathrm{L}^4_2$$

- **Linear operator = matrix-vector product**
  Program = matrix-vector product

$$\begin{bmatrix}1&1&1&1\\1&j&-1&-j\\1&-1&1&-1\\1&-j&-1&j\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\quad = ?\quad \texttt{DFT4([0,1,0,0])}$$

Symbolic evaluation and symbolic execution establishes correctness

Outline

- Introduction
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- **FFTX: A Library Frontend for SPIRAL**
- Summary

FFTX and SpectralPACK

Numerical Linear Algebra		Spectral Algorithms
LAPACK LU factorization Eigensolves SVD …	≈	**SpectralPACK** Convolution Correlation Upsampling Poisson solver …
BLAS BLAS-1 BLAS-2 BLAS-3		**FFTX** DFT, RDFT 1D, 2D, 3D,… batch

Define the LAPACK equivalent for spectral algorithms

- **Define FFTX as the BLAS equivalent**
  provide user FFT functionality as well as algorithm building blocks
- **Define class of numerical algorithms to be supported by SpectralPACK**
  PDE solver classes (Green's function, sparse in normal/k space,…), signal processing,…
- **Library front-end, code generation and vendor library back-end**
  mirror concepts from FFTX layer

FFTX and SpectralPACK solve the "spectral motif" long term

Example: Poisson's Equation in Free Space

Partial differential equation (PDE)

$$\Delta(\Phi) = \rho$$

$$\rho : \mathbb{R}^3 \to \mathbb{R}$$

$$D = \mathrm{supp}(\rho) \subset \mathbb{R}^3$$

Poisson's equation. Δ is the Laplace operator

Solution characterization

$$\Phi : \mathbb{R}^3 \to \mathbb{R}$$

$$\Phi(\vec{x}) = \frac{Q}{4\pi||\vec{x}||} + o\left(\frac{1}{||\vec{x}||}\right) \quad \text{as } ||\vec{x}|| \to \infty$$

$$Q = \int_D \rho d\vec{x}$$

Approach: Green's function

$$\Phi(\vec{x}) = \int_D G(\vec{x} - \vec{y})\rho(\vec{y})d\vec{y} \equiv (G * \rho)(\vec{x}), \quad G(\vec{x}) = \frac{1}{4\pi||\vec{x}||_2}$$

Solution: ϕ(.) = convolution of RHS *ρ*(.) with Green's function *G*(.). Efficient through FFTs (frequency domain)

Method of Local Corrections (MLC)

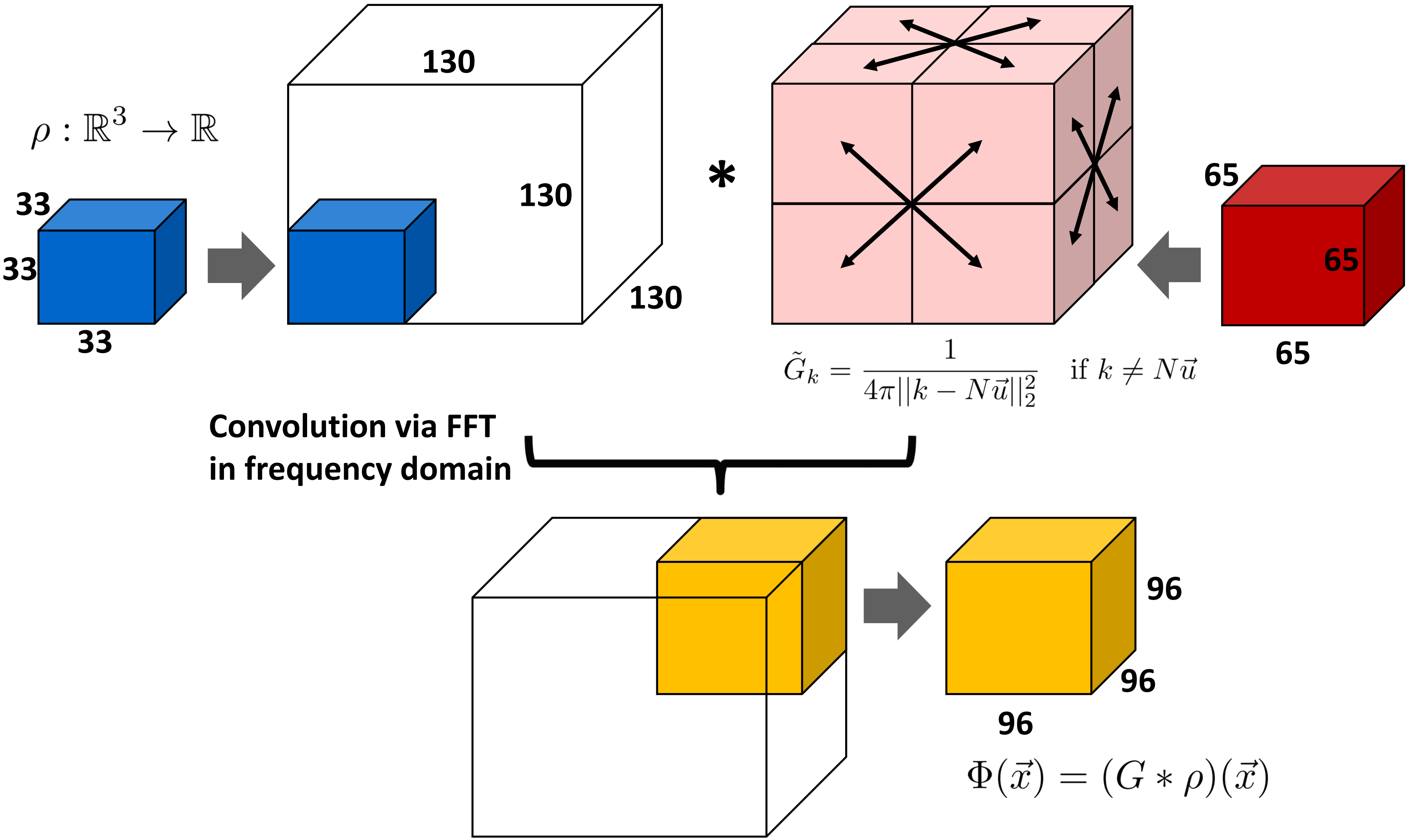
$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

Green's function kernel in frequency domain

P. McCorquodale, P. Colella, G. T. Balls, and S. B. Baden: **A Local Corrections Algorithm for Solving Poisson's Equation in Three Dimensions.** Communications in Applied Mathematics and Computational Science Vol. 2, No. 1 (2007), pp. 57-81., 2007.

C. R. Anderson: **A method of local corrections for computing the velocity field due to a distribution of vortex blobs.** Journal of Computational Physics, vol. 62, no. 1, pp. 111–123, 1986.

Algorithm: Hockney Free Space Convolution



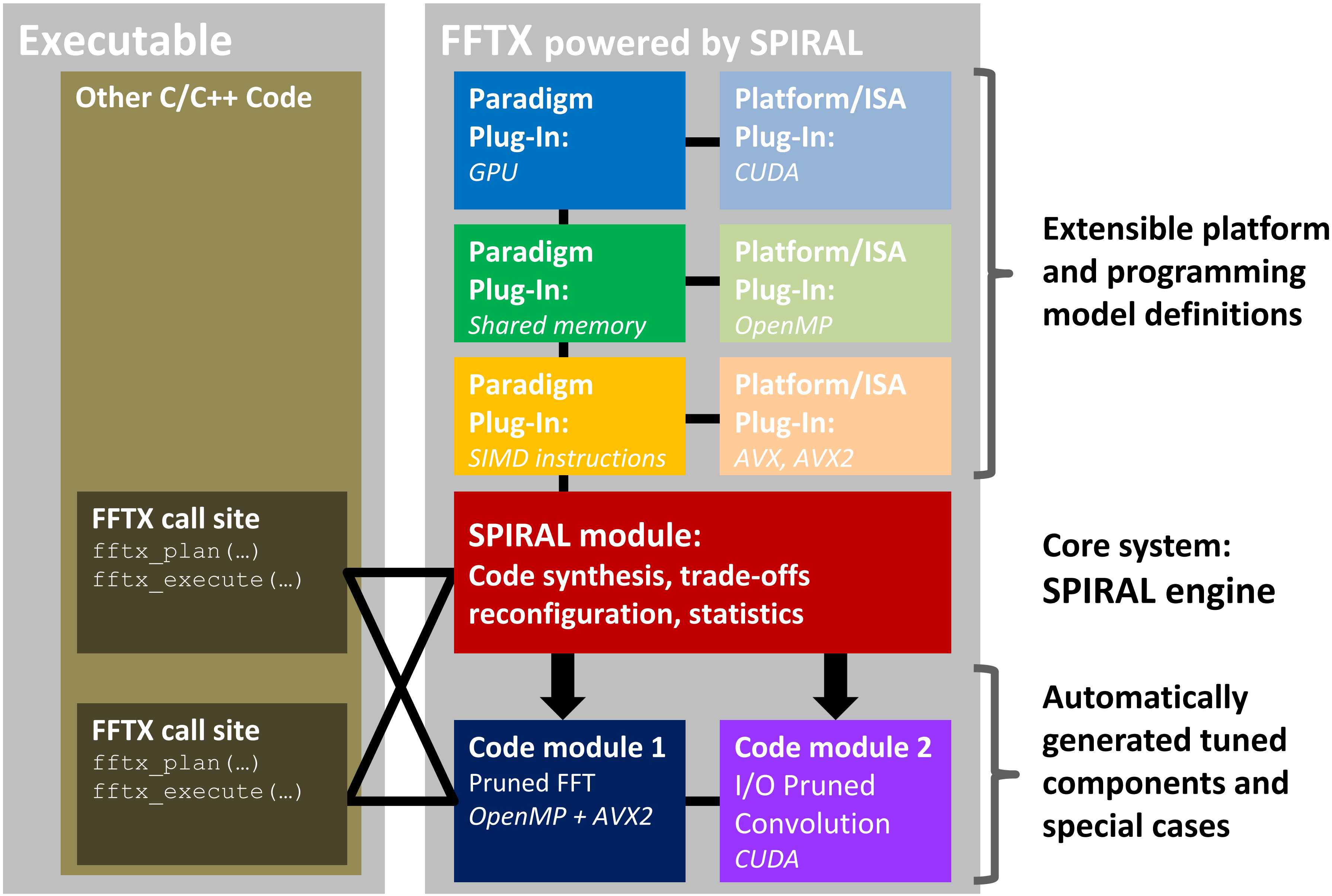
Hockney: Convolution + problem specific zero padding and output subset

FFTX C Code: Hockney Free Space Convolution

```
fftx_plan pruned_real_convolution_plan(fftx_real *in, fftx_real *out, fftx_complex *symbol,
        int n, int n_in, int n_out, int n_freq) {
    int rank = 3,
    batch_rank = 0,
    ...
    fftx_plan plans[5];
    fftx_plan p;

    tmp1 = fftx_create_zero_temp_real(rank, &padded_dims);

    plans[0] = fftx_plan_guru_copy_real(rank, &in_dimx, in, tmp1, MY_FFTX_MODE_SUB);

    tmp2 = fftx_create_temp_complex(rank, &freq_dims);
    plans[1] = fftx_plan_guru_dft_r2c(rank, &padded_dims, batch_rank,
        &batch_dims, tmp1, tmp2, MY_FFTX_MODE_SUB);

    tmp3 = fftx_create_temp_complex(rank, &freq_dims);
    plans[2] = fftx_plan_guru_pointwise_c2c(rank, &freq_dimx, batch_rank, &batch_dimx,
        tmp2, tmp3, symbol, (fftx_callback)complex_scaling,
        MY_FFTX_MODE_SUB | FFTX_PW_POINTWISE);

    tmp4 = fftx_create_temp_real(rank, &padded_dims);
    plans[3] = fftx_plan_guru_dft_c2r(rank, &padded_dims, batch_rank,
        &batch_dims, tmp3, tmp4, MY_FFTX_MODE_SUB);

    plans[4] = fftx_plan_guru_copy_real(rank, &out_dimx, tmp4, out, MY_FFTX_MODE_SUB);

    p = fftx_plan_compose(numsubplans, plans, MY_FFTX_MODE_TOP);

    return p;
}
```

Looks like FFTW calls, but is a specification for SPIRAL

FFTX Backend: SPIRAL



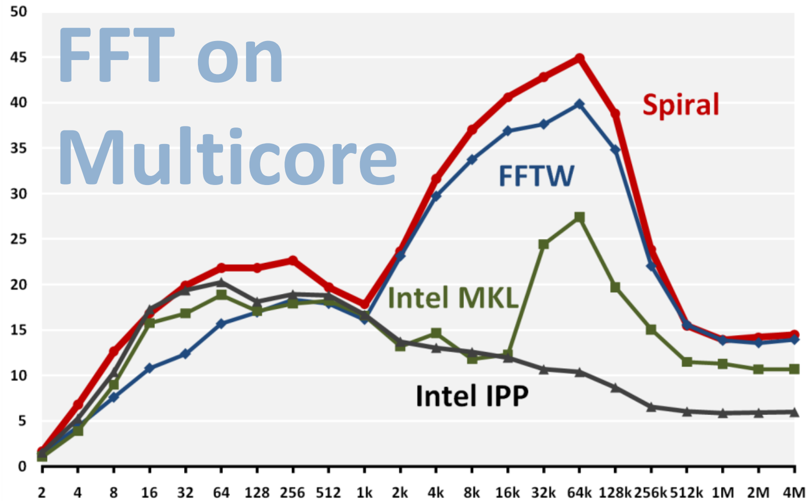
Outline

- Introduction
- Operator Language
- Achieving Performance Portability
- FFTX: A Library Frontend for SPIRAL
- **Summary**

Synthesis: FFTs and Spectral Algorithms

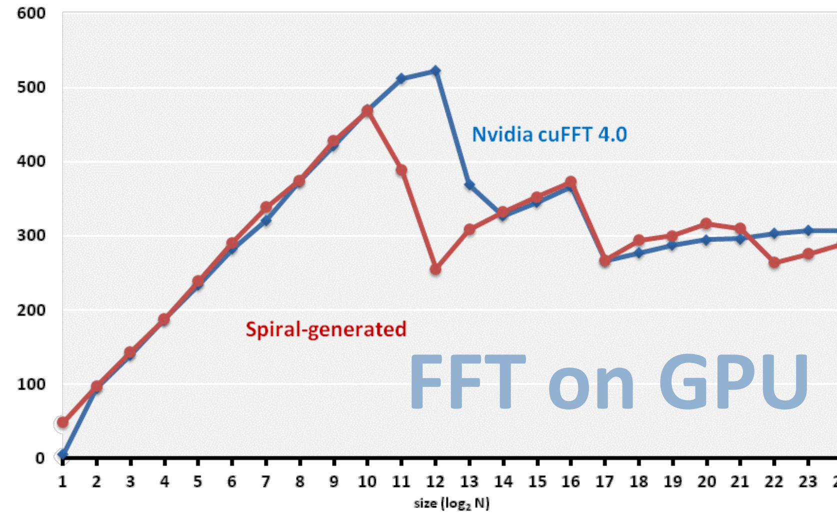
1D DFT on 3.3 GHz Sandy Bridge (4 Cores, AVX)

performance [Gflop/s]



1D Batch DFT (Nvidia GTX 480)

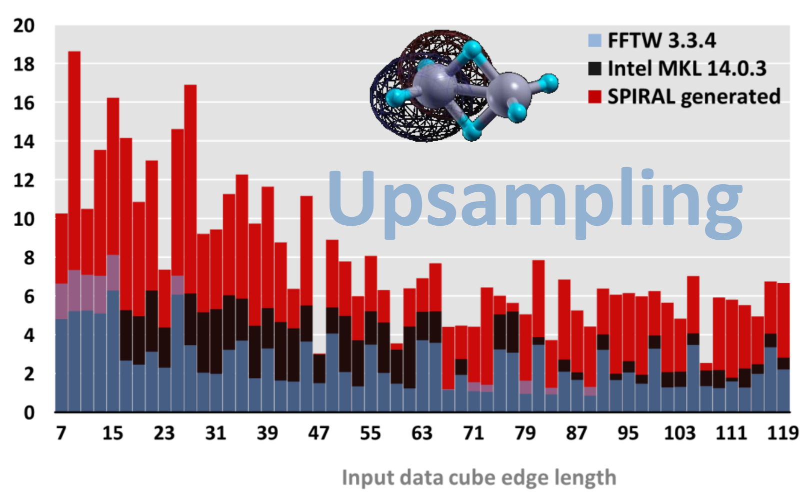
performance [GFlop/s] , single precision



Performance of 2x2x2 Upsampling on Haswell

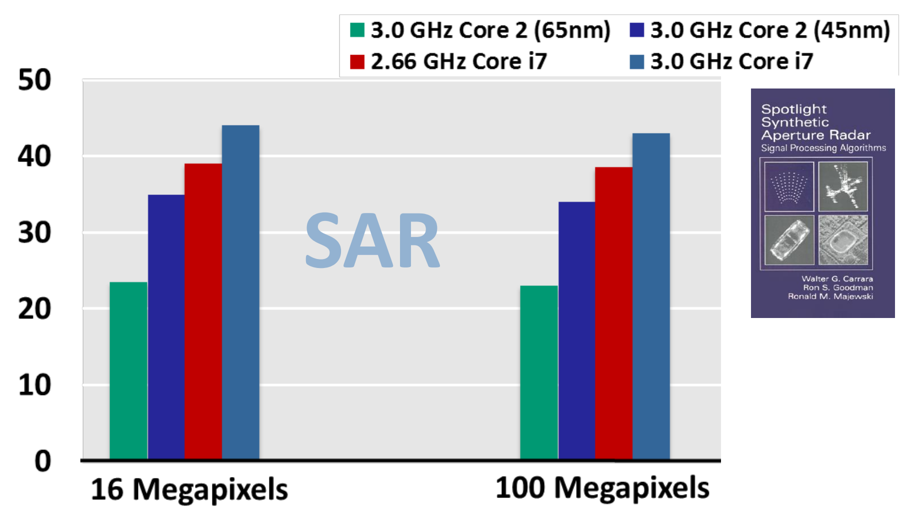
3.5 GHz, AVX, double precision, interleaved input, single core

Performance [Pseudo Gflop/s]



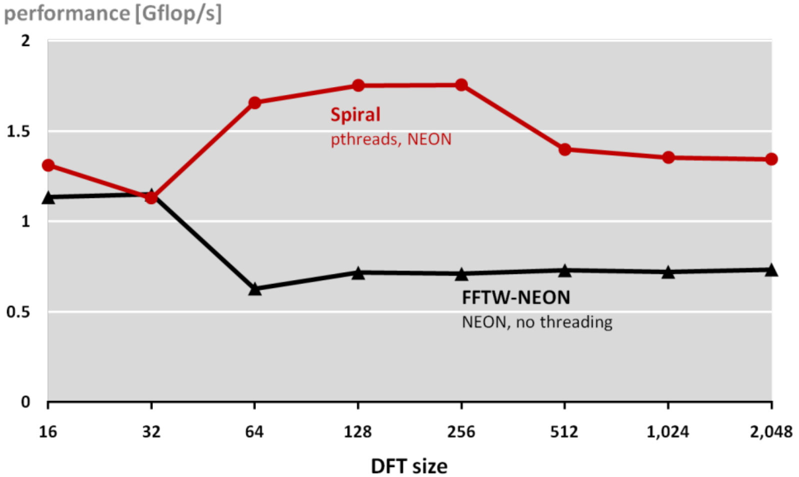
PFA SAR Image Formation on Intel platforms

performance [Gflop/s]

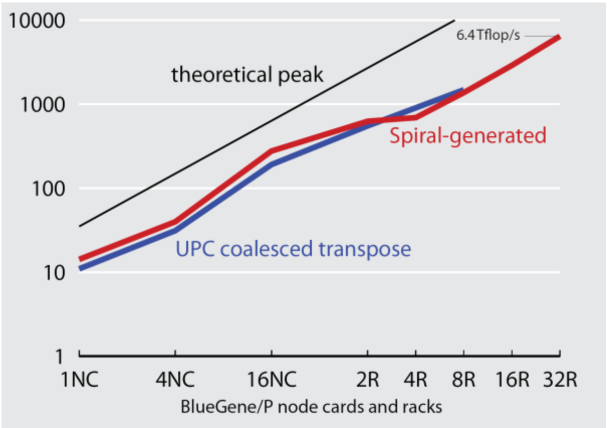


From Cell Phone To Supercomputer

DFT on Samsung Galaxy S II
Dual-core 1.2 GHz Cortex-A9 with NEON ISA



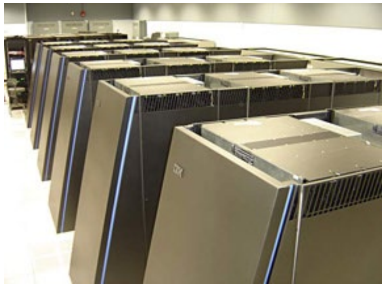
Global FFT (1D FFT, HPC Challenge)
performance [Gflop/s]



6.4 Tflop/s on BlueGene/P

Samsung i9100 Galaxy S II
Dual-core ARM at 1.2GHz with NEON ISA

BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz

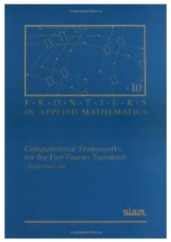


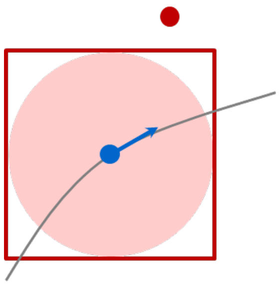
F. Gygi, E. W. Draeger, M. Schulz, B. R. de Supinski, J. A. Gunnels, V. Austel, J. C. Sexton, F. Franchetti, S. Kral, C. W. Ueberhuber, J. Lorenz, **"Large-Scale Electronic Structure Calculations of High-Z Metals on the BlueGene/L Platform,"** In Proceedings of Supercomputing, 2006. ***2006 Gordon Bell Prize (Peak Performance Award).***

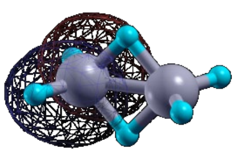
G. Almási, B. Dalton, L. L. Hu, F. Franchetti, Y. Liu, A. Sidelnik, T. Spelce, I. G. Tānase, E. Tiotto, Y. Voronenko, X. Xue, **"2010 IBM HPC Challenge Class II Submission,"** ***2010 HPC Challenge Class II Award (Most Productive System).***

SPIRAL: AI for High Performance Code

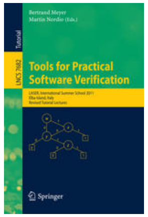
Algorithms

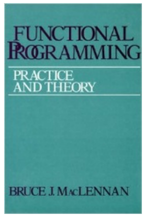


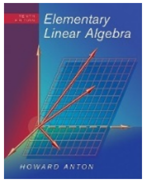




Correctness







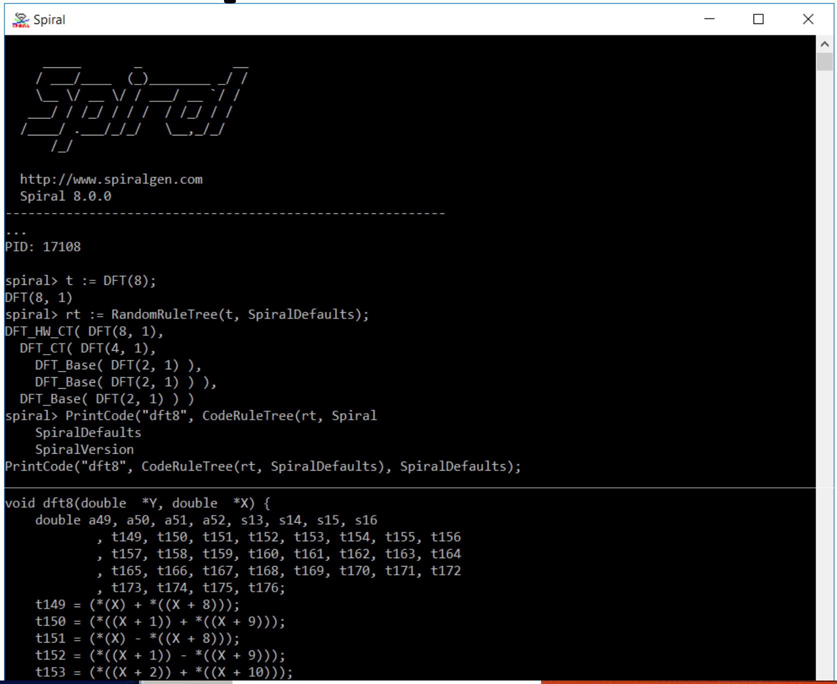

```
int dwmonitor(float  *X, double  *D) {
    __m128d u1, u2, u3, u4, u5, u6, u7, u8,...
    unsigned _xm = _mm_getcsr();
    _mm_setcsr(_xm & 0xffff0000 | 0x0000dfc0);
    u5 = _mm_set1_pd(0.0);
    u2 = _mm_cvtps_pd(_mm_addsub_ps(
    _mm_set1_ps(FLT_MIN), _mm_set1_ps(X[0])));
    u1 = _mm_set_pd(1.0, (-1.0));
    for(int i5 = 0; i5 <= 2; i5++) {
        x6 = _mm_addsub_pd(_mm_set1_pd((DBL_MIN
            +DBL_MIN)), _mm_loaddup_pd(&(D[i5])));
        x1 = _mm_addsub_pd(_mm_set1_pd(0.0), u1);
        x2 = _mm_mul_pd(x1, x6);
    ...
```

Hardware

SPIRAL 8.0: Available Under Open Source

- **Open Source SPIRAL available**
  - non-viral license (BSD)
  - Initial version, effort ongoing to open source whole system
  - Commercial support via SpiralGen, Inc.
- **Developed over 20 years**
  - Funding: DARPA (OPAL, DESA, HACMS, PERFECT, BRASS), NSF, ONR, DoD HPC, JPL, DOE, CMU SEI, Intel, Nvidia, Mercury
- **Open sourced under DARPA PERFECT**
- **First Tutorial @ HPEC 2019**

www.spiral.net



F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura: SPIRAL: Extreme Performance Portability, **Proceedings of the IEEE, Vol. 106, No. 11, 2018.**
Special Issue on *From High Level Specification to High Performance Code*